IM-098 · Vector geometry II: parallel vectors, collinear points and ratio

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 098

Extended only (E7.4.4). "Vector AB" means the vector from A to B (written with an arrow). In every "show that", write the vectors, the multiple, the reason (parallel, with a common point) and the conclusion in words.

Set A · Fluency

1. Show that 9a − 6b is parallel to 3a − 2b, and compare their lengths.

2. Which of these are parallel to 3a − 2b? For each one that is, give the scalar multiple. (i) −6a + 4b (ii) 3/2 a − b (iii) 3a + 2b

3. Find k so that ka + 8b is parallel to 5a + 2b.

4. Vector OA = a, vector OB = b and vector OC = 4b − 3a. Show that A, B and C are collinear, and find AB : BC.

5. OAB is a triangle with vector OA = a and vector OB = b. M and N are the midpoints of OA and OB. Show that MN is parallel to AB and find MN : AB.

6. a and b are not parallel. Find p and q if (p + 2)a + 3qb = 5a + (q + 4)b.

7. P, Q and R are collinear and vector PR = 4 × vector PQ. Find PQ : QR.

Set B · Exam style

1. (Paper 1/2 · no calculator) (E) OAB is a triangle with vector OA = a and vector OB = b. P is on OA with OP : PA = 1 : 2. Q is on AB with AQ : QB = 1 : 2.
   (a) Find vector OQ in terms of a and b. [2]
   (b) Find vector PQ in its simplest form. [2]
   (c) Show that PQ is parallel to the vector a + b, and find the ratio of the length of PQ to the length of a + b. [2]

2. (Paper 1/2 · no calculator) (E) OABC is a parallelogram with vector OA = a and vector OC = c. D is on the diagonal OB with OD : DB = 1 : 2. E is the midpoint of OA.
   (a) Find vector CD. [2]
   (b) Find vector CE. [1]
   (c) Show that C, D and E are collinear, and find CD : DE. [3]

3. (Paper 1/2 · no calculator) (E) Vector OA = a and vector OB = b. P is on AB and vector OP is parallel to 3a + b. Find vector OP and the ratio AP : PB. [4]

Challenge

(E, links to Ep 30) OAB is a triangle with vector OA = a and vector OB = b. M is the midpoint of OB, and L is on OA with vector OL = ⅓a. The lines AM and BL meet at G. Find vector OG and the ratio AG : GM.

Spaced review

1. (Ep 86) Solve $\sin x = 0.5$ for $0° \le x \le 360°$.

2. (Ep 75) Find the exact circumference of a circle of radius 7 cm.

Answers

Set A

1. $9\mathbf{a} - 6\mathbf{b} = 3(3\mathbf{a} - 2\mathbf{b})$: parallel, same direction, 3 times as long.

2. (i) $-2(3\mathbf{a} - 2\mathbf{b})$ ✓ (ii) $\frac{1}{2}(3\mathbf{a} - 2\mathbf{b})$ ✓ (iii) not parallel (coefficients 3 : 2, not 3 : −2).

3. $8\mathbf{b} = 4 \times 2\mathbf{b}$, so the vector is $4(5\mathbf{a} + 2\mathbf{b}) = 20\mathbf{a} + 8\mathbf{b}$: $k = 20$.

4. Vector AB = $\mathbf{b} - \mathbf{a}$; vector AC = $4\mathbf{b} - 3\mathbf{a} - \mathbf{a} = 4(\mathbf{b} - \mathbf{a}) = 4 \times$ vector AB. Parallel with the common point A, so A, B, C are collinear. BC = 3AB, so AB : BC = 1 : 3.

5. Vector MN = $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} = \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2} \times$ vector AB: parallel, MN : AB = 1 : 2.

6. $p + 2 = 5$ and $3q = q + 4$: $p = 3$, $q = 2$.

7. QR = PR − PQ = 3PQ, so PQ : QR = 1 : 3.

Set B

1. (a) Vector OQ = $\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$ M1 = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ A1.
   (b) Vector PQ = $-\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ M1 = $\frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ A1.
   (c) Vector PQ = $\frac{1}{3}(\mathbf{a} + \mathbf{b})$, a multiple of $\mathbf{a} + \mathbf{b}$, so parallel M1; ratio 1 : 3 A1.

2. (a) Vector OD = $\frac{1}{3}(\mathbf{a} + \mathbf{c})$ M1; vector CD = $-\mathbf{c} + \frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{c} = \frac{1}{3}\mathbf{a} - \frac{2}{3}\mathbf{c}$ A1.
   (b) Vector CE = $-\mathbf{c} + \frac{1}{2}\mathbf{a} = \frac{1}{2}\mathbf{a} - \mathbf{c}$ B1.
   (c) Vector CE = $3/2(\frac{1}{3}\mathbf{a} - \frac{2}{3}\mathbf{c}) = 3/2 \times$ vector CD M1; parallel with the common point C, so C, D, E are collinear A1; CD : DE = $1 : \frac{1}{2} = 2 : 1$ B1.

3. Vector OP = $\mathbf{a} + t(\mathbf{b} - \mathbf{a}) = (1 - t)\mathbf{a} + t\mathbf{b}$ M1; $= k(3\mathbf{a} + \mathbf{b})$ M1; $1 - t = 3k$ and $t = k$, so $k = t = \frac{1}{4}$ M1; vector OP = $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ and AP : PB = 1 : 3 A1.

Challenge

On AM: vector OG = $\mathbf{a} + \lambda(\frac{1}{2}\mathbf{b} - \mathbf{a}) = (1 - \lambda)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$. On BL: vector OG = $\mathbf{b} + \mu(\frac{1}{3}\mathbf{a} - \mathbf{b}) = \frac{1}{3}\mu\mathbf{a} + (1 - \mu)\mathbf{b}$. Equate: $1 - \lambda = \frac{1}{3}\mu$ and $\frac{1}{2}\lambda = 1 - \mu$, so $\lambda = 4/5$, $\mu = 3/5$. Vector OG = $\frac{1}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$, and AG : GM = 4/5 : 1/5 = 4 : 1.

Spaced review

1. $x = 30°$ or $x = 150°$.

2. $2\pi \times 7 = 14\pi$ cm.