IM-097 · Vector geometry I: expressing vectors in a shape

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 097

Extended only (E7.4.3). Arrows over two letters (e.g. AB) are written here as "vector AB". Draw a sketch for every question, write each route in letters first, and give every answer in simplest form in terms of the given vectors.

Set A · Fluency

1. Vector OA = a and vector OB = b. Write vector AB and vector BA in terms of a and b.

2. Vector OA = a and vector OB = b. M is the midpoint of AB. Find vector AM and vector OM.

3. OABC is a parallelogram with vector OA = a and vector OC = c. Find vector OB and vector AC.

4. OABC is a trapezium with vector OA = a, vector OC = c and vector CB = 3a. Find vector OB and vector AB.

5. P is on AB with AP : PB = 3 : 1. Write vector AP as a fraction of vector AB.

6. Vector OA = a and vector OB = b. P is on AB with AP : PB = 1 : 4. Find vector OP.

7. Simplify 2a + ½(b − 2a).

Set B · Exam style

1. (Paper 1/2 · no calculator) (E) OAB is a triangle with vector OA = a and vector OB = b. P is the midpoint of OB. Q is on AB with AQ : QB = 2 : 1.
   (a) Find vector OQ in terms of a and b. [2]
   (b) Find vector PQ in terms of a and b, in its simplest form. [2]

2. (Paper 1/2 · no calculator) (E) ABCDEF is a regular hexagon with vector AB = a and vector BC = b. The diagonal AD is parallel to BC and twice its length.
   (a) Find vector AC. [1]
   (b) Find vector AE. [2]
   (c) Find vector BE, and show that it is a multiple of vector CD. [2]

3. (Paper 1/2 · no calculator) (E) OABC is a parallelogram with vector OA = a and vector OC = c. D is on AB with AD : DB = 1 : 3. E is the midpoint of BC.
   (a) Find vector OD. [2]
   (b) Find vector OE. [1]
   (c) Find vector DE, in its simplest form. [2]

Challenge

(E, links to Ep 4 and Ep 13) Vector OA = a and vector OB = b. X is on OA with OX : XA = 1 : 2. Y is on AB with AY : YB = 3 : 2.
(a) Find vector OY.
(b) Find vector XY in its simplest form.
(c) Explain why vector XY is not parallel to vector OB, however long the sides are.

Answers

Set A

1. Vector AB = b − a; vector BA = a − b.

2. Vector AM = ½(b − a); vector OM = ½a + ½b = ½(a + b).

3. Vector OB = a + c; vector AC = c − a.

4. Vector OB = c + 3a; vector AB = −a + c + 3a = 2a + c.

5. 3/(3 + 1) = ¾ of vector AB.

6. a + ⅕(b − a) = ⅘a + ⅕b.

7. 2a + ½b − a = a + ½b.

Set B

1. (a) Vector AB = b − a M1; vector OQ = a + ⅔(b − a) = ⅓a + ⅔b A1.
   (b) Vector PQ = −½b + ⅓a + ⅔b M1 = ⅓a + ⅙b A1.

2. (a) a + b B1.
   (b) Vector AE = vector AD + vector DE = 2b + (−a) M1 = 2b − a A1.
   (c) Vector BE = −a + 2b − a = 2b − 2a M1; vector CD = b − a, so vector BE = 2(b − a) = 2 × vector CD A1.

3. (a) Vector AB = c, so vector AD = ¼c M1; vector OD = a + ¼c A1.
   (b) Vector OE = c + ½a B1 (vector CB = a).
   (c) Vector DE = −a − ¼c + c + ½a M1 = −½a + ¾c A1.

Challenge

(a) Vector OY = a + ⅗(b − a) = ⅖a + ⅗b.
(b) Vector XY = −⅓a + ⅖a + ⅗b = (2/5 − 1/3)a + ⅗b = 1/15 a + ⅗b.
(c) A vector parallel to vector OB is a multiple of b alone; vector XY has a non-zero multiple of a (1/15 a), so it cannot be parallel to b (a and b are not parallel).