IM-096 · Vectors II: magnitude, directed line segments and position vectors

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 096

Extended only (E7.3, E7.4.1–2). Column vectors are written here as (x over y). |a| means the magnitude of a. Give exact answers (surds in simplest form) on the non-calculator questions, and 3 significant figures on the calculator questions.

Set A · Fluency

1. Find the magnitude of (−9 over 12).

2. Find the exact magnitude of (−5 over 2).

3. Find the exact magnitude of (6 over −6), simplifying your surd.

4. (Calculator) Find the magnitude of (4.2 over 1.9), to 3 significant figures.

5. P is (4, −2) and Q is (−1, 3). Find PQ (arrow) as a column vector, and the exact length PQ.

6. A is (2, 7) and B is (8, −1). Find the position vector of the midpoint of AB.

7. The vector (k over 12) has magnitude 13. Find the possible values of k.

8. M(2, 0) is the midpoint of AB, and A is (−3, 4). Find the coordinates of B.

Set B · Exam style

1. (Paper 1/2 · no calculator) (E) A is (−1, 2) and B is (7, −4).
   (a) Write AB (arrow) as a column vector. [1]
   (b) Find |AB|. [2]
   (c) Find the position vector of the midpoint of AB. [2]

2. (Paper 1/2 · no calculator) (E) a = (3 over −4) and b = (−5 over 12).
   (a) Find |a| and |b|. [2]
   (b) Find |a + b|, giving your answer as a surd in its simplest form. [2]
   (c) Explain why |a + b| is less than |a| + |b|. [1]

3. (Paper 1/2 · no calculator) (E) A is (2, 1), B is (6, 4) and C is (−1, 5). Show that triangle ABC is isosceles and right-angled, and find its area. [4]

4. (Paper 3/4 · calculator) (E) P is (1.4, 3.3) and Q is (5.0, −4.4). Find the distance PQ. [2]

Challenge

(E, links to Ep 31) The vector v = (2t over t − 1) has magnitude √17.
(a) Show that $5t^2 - 2t - 16 = 0$, and find the two values of t.
(b) For the positive value of t, write down a vector parallel to v with magnitude 2√17.

Answers

Set A

1. $\sqrt{(81 + 144)} = \sqrt{225} = 15$.

2. $\sqrt{(25 + 4)} = \sqrt{29}$.

3. $\sqrt{72} = 6\sqrt{2}$.

4. $\sqrt{(17.64 + 3.61)} = \sqrt{21.25} = 4.6098\ldots \approx 4.61$.

5. $\overrightarrow{PQ} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$; $|PQ| = \sqrt{50} = 5\sqrt{2}$.

6. $\frac{1}{2}\left(\begin{pmatrix} 2 \\ 7 \end{pmatrix} + \begin{pmatrix} 8 \\ -1 \end{pmatrix}\right) = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

7. $k^2 + 144 = 169$, $k^2 = 25$, $k = 5$ or -5.

8. $\mathbf{b} = 2\mathbf{m} - \mathbf{a} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \end{pmatrix}$, so B is $(7, -4)$.

Set B

1. (a) $\begin{pmatrix} 8 \\ -6 \end{pmatrix}$ B1. (b) $\sqrt{(8^2 + (-6)^2)}$ M1 $= 10$ A1. (c) $\frac{1}{2}(\mathbf{a} + \mathbf{b})$ M1 $= \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ A1.

2. (a) $|\mathbf{a}| = \sqrt{9 + 16} = 5$, $|\mathbf{b}| = \sqrt{25 + 144} = 13$ B1 B1.
   (b) $\mathbf{a} + \mathbf{b} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$ M1; $\sqrt{4 + 64} = \sqrt{68} = 2\sqrt{17}$ A1.
   (c) Going along a then b is a longer route than going straight along $\mathbf{a} + \mathbf{b}$ (the two vectors are not in the same direction) B1. ($2\sqrt{17} \approx 8.25 < 18$.)

3. $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} -7 \\ 1 \end{pmatrix}$ M1; $|AB| = |AC| = 5$, so isosceles A1; $|BC|^2 = 50 = 25 + 25 = |AB|^2 + |AC|^2$, so the angle at A is 90° (converse of Pythagoras) A1; area $= \frac{1}{2} \times 5 \times 5 = 12.5$ B1.

4. $\overrightarrow{PQ} = \begin{pmatrix} 3.6 \\ -7.7 \end{pmatrix}$ M1; $\sqrt{(12.96 + 59.29)} = \sqrt{72.25} = 8.5$ A1 (8.50 to 3 s.f.).

Challenge

(a) $(2t)^2 + (t - 1)^2 = 17$, so $4t^2 + t^2 - 2t + 1 = 17$, giving $5t^2 - 2t - 16 = 0$. $(5t + 8)(t - 2) = 0$, so $t = 2$ or $t = -8/5$.
(b) $t = 2$ gives $\mathbf{v} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $|\mathbf{v}| = \sqrt{17}$. $2\mathbf{v} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ has magnitude $2\sqrt{17}$.