IM-094 · Combined transformations

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 094

Extended only (E7.1: questions may involve combinations of transformations). Draw every answer on squared paper with axes from −6 to 6, rule every straight edge, and label each image. Column vectors are written here as (a over b).

Set A · Fluency

1. (E) Reflect the triangle (2, 1), (5, 1), (2, 3) in the y-axis, then translate the image by (1 over −4). Write down the vertices of the final image.

2. (E) Rotate the triangle (1, 1), (3, 1), (3, 2) through 90° anticlockwise about (0, 0), then reflect the image in the x-axis. Write down the vertices of the final image.

3. (E) Do question 2 in the other order: reflect in the x-axis first, then rotate through 90° anticlockwise about (0, 0). Is the final image the same?

4. (E) Describe fully the single transformation equivalent to a reflection in x = 1 followed by a reflection in x = 4.

5. (E) Describe fully the single transformation equivalent to a reflection in the x-axis followed by a reflection in the line y = x.

6. (E) Describe fully the single transformation equivalent to a translation by (2 over −5) followed by a translation by (−7 over 1).

7. (E) Describe fully the inverse of (a) a translation by (−3 over 8) (b) a rotation through 90° clockwise about (1, 4) (c) an enlargement, scale factor 4, centre (2, 0). (d) Write down the invariant points of a reflection in the line y = −x.

Set B · Exam style

1. (Paper 1/2 · no calculator) (E) Triangle A has vertices (1, 1), (4, 1) and (1, 2).
   (a) Reflect triangle A in the line x = 2. Label the image B. [2]
   (b) Reflect triangle B in the x-axis. Label the image C. [2]
   (c) Describe fully the single transformation that maps triangle A onto triangle C. [3]

2. (Paper 1/2 · no calculator) (E) Triangle P has vertices (2, 1), (4, 1) and (2, 4). It is enlarged by scale factor 2, centre (0, 0), and the image is then translated by (−3 over 1).
   (a) Find the vertices of the final image. [2]
   (b) The final image is also an enlargement of P with scale factor 2. Find its centre. [2]

3. (Paper 3/4 · calculator) (E) M is a reflection in the line y = x. R is a rotation through 90° clockwise about (0, 0).
   (a) Find the image of the point (5, 2) under M followed by R. [2]
   (b) Describe fully the single transformation equivalent to M followed by R. [2]
   (c) Write down all the invariant points of this single transformation. [1]

Challenge

(E, links to Ep 92) A shape is reflected in the line y = 1 and then the image is reflected in the line x = 3.

(a) Show that the combination is equivalent to a rotation of 180° about (3, 1).

(b) A point P is mapped onto (7, −4) by the combination. Find the coordinates of P.

(c) Explain why doing the combination twice brings every point back to where it started.

Spaced review

1. (Ep 50) Find the exact length of the line segment joining (−1, 6) and (7, −4). Give your answer as a surd in its simplest form.

2. (Ep 16) Increase 240 by 15%.

Answers

Set A

1. After the reflection: (−2, 1), (−5, 1), (−2, 3). Final: (−1, −3), (−4, −3), (−1, −1).

2. After the rotation: (−1, 1), (−1, 3), (−2, 3). Final: (−1, −1), (−1, −3), (−2, −3).

3. After the reflection: (1, −1), (3, −1), (3, −2). Final: (1, 1), (1, 3), (2, 3). No: the order matters.

4. Translation by (6 over 0) (parallel mirrors 3 apart).

5. (x, y) → (x, −y) → (−y, x): rotation, 90° anticlockwise, centre (0, 0).

6. Translation by (−5 over −4).

7. (a) Translation by (3 over −8). (b) Rotation, 90° anticlockwise, centre (1, 4). (c) Enlargement, scale factor ¼, centre (2, 0). (d) Every point on the line y = −x.

Set B

1. (a) B: (3, 1), (0, 1), (3, 2) B2. (b) C: (3, −1), (0, −1), (3, −2) B2.
(c) Rotation B1, 180° B1, centre (2, 0) B1 (perpendicular mirrors x = 2 and y = 0 meet at (2, 0); check: the midpoint of (1, 1) and (3, −1) is (2, 0)).

2. (a) Enlarge: (4, 2), (8, 2), (4, 8) M1; translate: (1, 3), (5, 3), (1, 9) A1.
(b) Centre c with c + 2(p − c) = 2p + (−3, 1), so −c = (−3, 1) M1; centre (3, −1) A1. (Check: from (3, −1), (2, 1) is −1, 2; doubled −2, 4 gives (1, 3).)

3. (a) M: (2, 5) M1; R: (5, −2) A1.
(b) (x, y) → (y, x) → (x, −y): reflection B1 in the x-axis (y = 0) B1.
(c) Every point on the x-axis B1.

Challenge

(a) (x, y) → (x, 2 − y) → (6 − x, 2 − y). A rotation of 180° about (3, 1) sends (x, y) to (2 × 3 − x, 2 × 1 − y) = (6 − x, 2 − y): the same. (The mirrors are perpendicular and meet at (3, 1).)
(b) The half turn is its own inverse: P = (6 − 7, 2 − (−4)) = (−1, 6). Check: (−1, 6) → (−1, −4) → (7, −4).
(c) Two half turns about the same point make a full turn of 360°, so every point returns to its start.

Spaced review

1. $\sqrt{(8^2 + (-10)^2)} = \sqrt{164} = 2\sqrt{41}$.

2. 240 × 1.15 = 276.