IM-093 · Translation and enlargement

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 093

Core + Extended (C7.1.3–4, E7.1.3–4). Questions tagged (E) are Extended only. Draw every answer on squared paper, rule every straight edge, label each image, and write column vectors with no fraction line.

Set A · Fluency

1. Translate the triangle with vertices (1, 2), (4, 2) and (1, 3) by the vector (3 over −4), written as a column vector with 3 on top. Write down the vertices of the image.

2. A translation maps the point (−2, 5) onto the point (3, 1). Write down the column vector of the translation.

3. Triangle A has vertices (2, 3), (4, 3), (2, 6). Triangle B has vertices (−1, 1), (1, 1), (−1, 4). Describe fully the single transformation that maps A onto B.

4. Enlarge the triangle with vertices (1, 1), (3, 1) and (1, 2) by scale factor 3, centre (0, 0).

5. Enlarge the triangle with vertices (2, 2), (3, 2) and (2, 4) by scale factor 2, centre (1, 1).

6. Enlarge the triangle with vertices (4, 2), (8, 2) and (4, 6) by scale factor ½, centre (0, 0).

7. (a) A side of 3 cm on a shape is 7.5 cm on its image after an enlargement. Write down the scale factor.
   (b) Triangle P has vertices (1, 2), (2, 2), (1, 3); triangle Q has vertices (1, 2), (4, 2), (1, 5). Describe fully the single transformation that maps P onto Q.

8. (E) Enlarge the triangle with vertices (1, 1), (2, 1) and (1, 3) by scale factor −1, centre (0, 0). Which single rotation has the same effect?

Set B · Exam style

1. (Paper 1/2 · no calculator) (C) Shape S has vertices (−4, 1), (−2, 1), (−2, 2) and (−4, 3).
   (a) Draw the image of shape S after a translation by the column vector (6 over −3). [2]
   (b) Write down the column vector of the translation that maps the image back onto S. [1]

2. (Paper 1/2 · no calculator) (C) Triangle A has vertices (1, 1), (3, 1) and (1, 4).
   (a) Draw the image of triangle A after an enlargement, scale factor 2, centre (−1, 0). [2]
   (b) Triangle C has vertices (5, 1), (7, 1) and (5, 4). Describe fully the single transformation that maps triangle A onto triangle C. [2]

3. (Paper 1/2 · no calculator) (C) Triangle P has vertices (3, 3), (9, 3) and (3, 9). Triangle Q has vertices (−1, 1), (1, 1) and (−1, 3). Describe fully the single transformation that maps triangle P onto triangle Q. [3]

4. (Paper 1/2 · no calculator) (E) Triangle D has vertices (4, 2), (8, 2) and (4, 6).
   (a) Draw the image of triangle D after an enlargement, scale factor −½, centre (0, 0). Label it E. [2]
   (b) Write down the value of (area of E) ÷ (area of D). [1]
   (c) Describe fully the single transformation that maps triangle E onto triangle D. [2]

Challenge

(E, links to Ep 68 and Ep 69) Triangle T has vertices (1, 1), (4, 1) and (1, 3). It is enlarged with centre (−2, −1) and scale factor k. One vertex of the image is (7, 5).

(a) Which vertex of T maps to (7, 5)? Explain how you know, and find k.

(b) Find the other two vertices of the image.

(c) Find the area of T and the area of the image.

(d) A different enlargement with the same centre maps (1, 1) onto (−5, −3). Find its scale factor.

Answers

Set A

1. (4, −2), (7, −2), (4, −1).

2. (5 over −4).

3. Translation by the vector (−3 over −2).

4. (3, 3), (9, 3), (3, 6).

5. (3, 3), (5, 3), (3, 7).

6. (2, 1), (4, 1), (2, 3).

7. (a) 7.5 ÷ 3 = 2.5. (b) Enlargement, scale factor 3, centre (1, 2).

8. (−1, −1), (−2, −1), (−1, −3). The same as a rotation of 180° about (0, 0).

Set B

1. (a) (2, −2), (4, −2), (4, −1), (2, 0) B2 (B1 for a translation correct in one direction only).
   (b) (−6 over 3) B1.

2. (a) (3, 2), (7, 2), (3, 8) B2 (B1 for the correct size and orientation in the wrong place, or two vertices correct).
   (b) Translation B1, by (4 over 0) B1.

3. Enlargement B1, scale factor ⅓ B1 (6 → 2), centre (−3, 0) B1 (lines joining matching vertices meet at (−3, 0)).

4. (a) E: (−2, −1), (−4, −1), (−2, −3) B2 (B1 for scale factor +½ or −2 used correctly).
   (b) ¼ B1 ($(-½)^2 = ¼$).
   (c) Enlargement B1, scale factor −2, centre (0, 0) B1.

Challenge

(a) From (−2, −1), the image vertex is 9 right and 6 up. Only (1, 1) is a matching direction: it is 3 right and 2 up, and 9 = 3 × 3, 6 = 3 × 2. (For (4, 1) the arm is 6, 2 and for (1, 3) it is 3, 4: neither is in the ratio 9 : 6.) So k = 3.
(b) (4, 1): −2 + 3 × 6 = 16, −1 + 3 × 2 = 5, so (16, 5). (1, 3): −2 + 3 × 3 = 7, −1 + 3 × 4 = 11, so (7, 11).
(c) Area of T = ½ × 3 × 2 = 3. Area of image = 3^2 × 3 = 27.
(d) (−5, −3) is 3 left and 2 down from (−2, −1), the opposite of 3 right and 2 up, so the scale factor is −1.