IM-091 · 3D trigonometry II: the angle between a line and a plane

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 091 · Extended

Set A · Fluency

(This sheet is Extended (E6.6) throughout, except the spaced-review questions. The angle between a line and a plane is the angle between the line and its projection on the plane. Cuboid ABCDEFGH has base ABCD and top EFGH, with E above A, F above B, G above C and H above D. Give angles to 1 decimal place unless exact.)

1. (E) A cuboid is 10 cm by 6 cm by 4 cm (height 4 cm). Find the angle between a diagonal of the cuboid and its base.

2. (E) In cuboid ABCDEFGH, AB = 9 cm, BC = 12 cm and AE = 8 cm. Find the angle between AG and the base ABCD.

3. (E) In the cuboid of question 2, find the angle between AH and the base ABCD.

4. (E) Without a calculator: ABCDEFGH is a cube. (a) Write down the angle between the face diagonal AF and the base ABCD. (b) Explain why the angle between the diagonal AG and the base is less than 45°.

5. (E) A square-based pyramid has base edges 6 cm, and its top V is 8 cm vertically above the centre of the base. Find the angle between a slant edge and the base.

6. (E) For the pyramid in question 5, find the angle between a triangular face and the base.

7. (E) A straight pole 4.2 m long is pushed into level ground at a slant, and its top is 3.1 m above the ground. Find the angle between the pole and the ground.

8. (E) A wedge has a horizontal rectangular base ABCD with AB = 16 cm and BC = 6 cm, and a vertical rectangular end BCFE with BE = 7 cm. Find the angle between AF and the base.

Set B · Exam style

1. (Paper 4 · calculator) (E) In cuboid ABCDEFGH, AB = 11 cm, BC = 7 cm and CG = 4 cm.
   (a) Calculate AC. [2]
   (b) Calculate the angle between AG and the base ABCD. [2]
   (c) Calculate the angle between AG and the face BCGF. [3]

2. (Paper 4 · calculator) (E) Pyramid VABCD has a square base of side 9 cm. V is 11 cm vertically above the centre M of the base.
   (a) Calculate VA. [3]
   (b) Calculate the angle between VA and the base. [2]
   (c) Calculate the angle between the face VAB and the base. [2]

3. (Paper 4 · calculator) (E) A vertical flagpole FT, 14 m tall, stands at the corner F of a horizontal rectangular courtyard FGHK with FG = 30 m and GH = 22 m.
   (a) Calculate the angle of elevation of T from G. [2]
   (b) Calculate the angle between the line HT and the courtyard. [3]

4. (Paper 4 · calculator) (E) A tent has a horizontal rectangular base ABCD with AB = 3 m and BC = 2.4 m. The ridge PQ is horizontal and 1.6 m above the ground, with P vertically above the midpoint of AD and Q vertically above the midpoint of BC.

(a) Calculate the angle between the sloping face ABQP and the ground. [2]

(b) Calculate the angle between the line AQ and the ground. [3]

Challenge

(E) ABCDEFGH is a cube with edge 1 unit (links Ep 84 and Ep 90).

(a) Show that the angle between the diagonal AG and the base is $\tan^{-1}(1/\sqrt{2})$.

(b) Show that the angle between AG and the edge AE is $\cos^{-1}(1/\sqrt{3})$, and find it to 1 d.p.

Spaced review

1. (Ep 49 · Paper 1/2) Find the gradient of the line through (2, −1) and (6, 7).

2. (Ep 16 · Paper 1/2) Increase 80 kg by 15%.

Answers

Set A

1. Projection = base diagonal = $\sqrt{(100 + 36)} = \sqrt{136}$; $\tan\theta = 4/\sqrt{136}$, $\theta = 18.9°$.

2. AC = 15; $\tan GAC = 8/15$, angle = 28.1°.

3. H is above D, so the projection of AH is AD = 12; $\tan HAD = 8/12$, angle = 33.7°.

4. (a) F is above B, so the projection of AF is AB and the angle is FAB = 45° (AB = BF). (b) The projection AC is longer than the height CG ($AC = \sqrt{2} \times$ edge), so $\tan GAC = 1/\sqrt{2} < 1$ and the angle is less than 45° (it is 35.3°).

5. Half the base diagonal = $\frac{1}{2}\sqrt{72} = 3\sqrt{2} = 4.242…$; $\tan\theta = 8/(3\sqrt{2})$, $\theta = 62.1°$.

6. From the centre to the midpoint of an edge is 3; $\tan\theta = 8/3$, $\theta = 69.4°$.

7. $\sin\theta = 3.1/4.2$, $\theta = 47.6°$.

8. F is above C: projection $AC = \sqrt{(256 + 36)} = \sqrt{292} = 17.08…$; $\tan\theta = 7/\sqrt{292}$, $\theta = 22.3°$.

Set B

1. (a) $AC = \sqrt{(121 + 49)}$ M1 $= \sqrt{170} = 13.03… = 13.0$ cm A1.
   (b) $\tan GAC = 4/\sqrt{170}$ M1; angle = 17.1° A1.
   (c) AB ⊥ face BCGF, so the projection of AG is BG B1; $BG = \sqrt{(49 + 16)} = \sqrt{65}$, $\tan AGB = 11/\sqrt{65}$ M1; angle AGB = 53.8° A1.

2. (a) $AM = \frac{1}{2}\sqrt{(81 + 81)} = 4.5\sqrt{2} = 6.363…$ M1; $VA = \sqrt{(AM^2 + 11^2)}$ M1 $= 12.70… = 12.7$ cm A1.
   (b) $\tan VAM = 11/(4.5\sqrt{2})$ M1; angle = 59.9° A1.
   (c) N the midpoint of AB, MN = 4.5; $\tan VNM = 11/4.5$ M1; angle = 67.8° A1.

3. (a) $\tan TGF = 14/30$ M1; angle = 25.0° A1.
   (b) Projection of HT is HF B1; $HF = \sqrt{(30^2 + 22^2)} = \sqrt{1384} = 37.20…$ M1; $\tan THF = 14/37.20…$, angle = 20.6° A1.

4. (a) The line in the face at right angles to AB goes from the midpoint of AB to the midpoint of PQ; its projection is half of BC = 1.2 m. $\tan\theta = 1.6/1.2$ M1; $\theta = 53.1°$ A1.
   (b) Q is above the midpoint N of BC; projection $AN = \sqrt{(3^2 + 1.2^2)} = \sqrt{10.44} = 3.231…$ M1 A1; $\tan QAN = 1.6/3.231…$, angle = 26.3° A1.

Challenge

(a) Projection of AG on the base is $AC = \sqrt{2}$; GC = 1; $\tan GAC = 1/\sqrt{2}$, so the angle is $\tan^{-1}(1/\sqrt{2})$ (= 35.3°).
(b) Triangle AEG has its right angle at E (EG lies in the top face, AE is vertical). $AG = \sqrt{3}$, AE = 1, so $\cos GAE = 1/\sqrt{3}$ and the angle is 54.7°.

Spaced review

1. $(7 - (-1))/(6 - 2) = 8/4 = 2$.

2. $80 \times 1.15 = 92$ kg.