IM-080 · Compound solids, frustums and parts of solids

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 080 · Core with an Extended section

All solid formulas used here (cylinder, cone, sphere, pyramid, prism) are on the formula page. Frustum: big cone − small cone (no formula given). Sector area and arc length must be learnt.

Set A · Fluency

1. (C) A solid is made from a cylinder of radius 4 cm and height 10 cm with a hemisphere of radius 4 cm on top. Find its volume in terms of π.

2. (C) For the solid in question 1, find the total surface area in terms of π.

3. (C) A cuboid 5 cm by 5 cm by 8 cm has a square-based pyramid of height 6 cm on its 5 cm by 5 cm top face. Find the volume of the solid.

4. (C) A solid hemisphere has radius 5 cm. Find (a) its volume (b) its total surface area, in terms of π.

5. (C) A solid half cylinder has radius 6 cm and length 20 cm. Find its total surface area in terms of π.

6. (E) A cone has base radius 9 cm and height 12 cm. A small cone of radius 3 cm and height 4 cm is cut off the top. Find the volume of the frustum that is left, in terms of π.

7. (E) A solid metal cube of side 10 cm is melted and recast into spheres of radius 2 cm. How many complete spheres can be made?

8. (E) A cylinder of radius 10 cm contains water. A solid metal cube of side 5 cm is dropped in and is completely covered. By how much does the water level rise? Give your answer to 3 s.f.

Set B · Exam style

1. (Paper 1/2 · no calculator) (C) A solid is made from a cone on top of a cylinder. Both have radius 6 cm. The cylinder has height 5 cm. The cone has height 8 cm and slant height 10 cm.
   (a) Find the volume of the solid in terms of π. [3]
   (b) Find the total surface area of the solid in terms of π. [3]

2. (Paper 3/4 · calculator) (C) A wedge is a prism whose cross-section is a sector of a circle of radius 12 cm and angle 30°. The wedge is 4 cm thick.
   (a) Calculate the volume of the wedge. [2]
   (b) Calculate the total surface area of the wedge. [3]

3. (Paper 3/4 · calculator) (E) A cup is a frustum of a cone. The top has radius 4.5 cm, the base has radius 3 cm and the cup is 9 cm deep.
   (a) Show that the height of the complete cone is 27 cm. [2]
   (b) Calculate the capacity of the cup in millilitres. (1 cm^3 = 1 ml) [3]

4. (Paper 1/2 · no calculator) (E) A solid metal hemisphere of radius 6 cm is melted down and recast into a solid cone with base radius 6 cm. Find the height of the cone. [3]

Challenge

(E) A cone-shaped container stands with its point down. It has height 20 cm and top radius 8 cm. Water is poured in to a depth of 10 cm.

(a) What fraction of the container's volume is filled with water? Explain your answer.

(b) The water is poured into an empty cylinder of radius 4 cm. How deep is the water in the cylinder? Give your answer as an exact fraction.

(Links to Ep 69, similar volumes.)

Answers

Set A

1. $160\pi + (128/3)\pi = (608/3)\pi$ cm³

2. $16\pi + 80\pi + 32\pi = 128\pi$ cm²

3. $200 + 50 = 250$ cm³

4. (a) $(250/3)\pi$ cm³ (b) 75π cm²

5. $36\pi + 120\pi + 240 = 156\pi + 240$ cm²

6. $\frac{1}{3}\pi(81)(12) - \frac{1}{3}\pi(9)(4) = 324\pi - 12\pi = 312\pi$ cm³

7. One sphere = $(32/3)\pi = 33.51\ldots$ cm³; $1000 \div 33.51\ldots = 29.8\ldots$, so 29 spheres.

8. $100\pi d = 125$, $d = 0.398$ cm

Set B

1. (a) Cylinder $\pi(36)(5) = 180\pi$ M1; cone $\frac{1}{3}\pi(36)(8) = 96\pi$ M1; 276π cm³ A1.
   (b) base 36π, cylinder side $2\pi(6)(5) = 60\pi$, cone $\pi(6)(10) = 60\pi$ M2 (M1 for two of the three correct); 156π cm² A1.

2. (a) $30/360 \times \pi \times 144 \times 4$ M1 $= 48\pi = 151$ cm³ A1.
   (b) Two sectors $24\pi = 75.39\ldots$; curved face $(2\pi) \times 4 = 25.13\ldots$; two rectangles $2 \times 12 \times 4 = 96$ M2 (M1 for two kinds of face); 197 cm² A1 (196.5…).

3. (a) $h/(h + 9) = 3/4.5$ M1; $4.5h = 3h + 27$, $h = 18$, so $18 + 9 = 27$ cm A1.
   (b) $\frac{1}{3}\pi(4.5)^2(27) - \frac{1}{3}\pi(3)^2(18)$ M1 $= 182.25\pi - 54\pi = 128.25\pi$ M1; 403 ml A1 (402.9…).

4. Hemisphere $\frac{2}{3}\pi(216) = 144\pi$ B1; $\frac{1}{3}\pi(36)h = 144\pi$ M1; $12h = 144$, $h = 12$ cm A1.

Challenge

(a) The water is a cone similar to the container with scale factor ½, so its volume is $(\frac{1}{2})^3 = \frac{1}{8}$ of the container.
(b) Water = $\frac{1}{3}\pi(4)^2(10) = (160/3)\pi$ cm³ (radius 4 at depth 10); $16\pi d = (160/3)\pi$, $d = 10/3$ cm (3.33 cm).