IM-069 · Similar shapes: areas and volumes (Extended)

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 069

All questions are Extended (E4.4). Lengths × k, areas × k^2, volumes × k^3.

Set A · Fluency

1. (E) Two similar solids have lengths in the ratio 1 : 3. Write down the ratio of (a) their surface areas (b) their volumes.

2. (E) Two similar rectangles have widths 3 cm and 12 cm. The small one has area 15 cm^2. Find the area of the large one.

3. (E) Two similar triangles have areas 20 cm^2 and 45 cm^2. The shortest side of the small triangle is 6 cm. Find the shortest side of the large triangle.

4. (E) A map has a scale of 1 : 20 000. A field covers 4.5 cm^2 on the map. Find the real area of the field in km^2.

5. (E) Two similar jars have heights 6 cm and 9 cm. The small jar holds 160 ml. How much does the large jar hold?

6. (E) Two similar solids have volumes 27 cm^3 and 125 cm^3. The large solid is 10 cm tall. How tall is the small one?

7. (E) Two similar solids have surface areas in the ratio 4 : 9. Find the ratio of their volumes.

8. (E) Two similar statues are made of the same metal. Their masses are 2 kg and 16 kg. The small statue is 30 cm tall. How tall is the large one?

Set B · Exam style

1. (Paper 1/2 · no calculator) (E) Two similar cylinders have heights 8 cm and 12 cm. The large cylinder has surface area 360 cm^2. The small cylinder has volume 96 cm^3.
   (a) Find the surface area of the small cylinder. [2]
   (b) Find the volume of the large cylinder. [2]

2. (Paper 3/4 · calculator) (E) Two similar bottles hold 330 ml and 500 ml. The small bottle is 14 cm tall. Calculate the height of the large bottle. [3]

3. (Paper 1/2 · no calculator) (E) In triangle ABC, D is on AB and E is on AC, with DE parallel to BC. AD : DB = 2 : 3. The area of triangle ADE is 12 cm^2.
   (a) Show that triangles ADE and ABC are similar. [2]
   (b) Find the area of triangle ABC. [2]
   (c) Find the area of the trapezium DBCE. [1]

4. (Paper 3/4 · calculator) (E) Two similar solid cones are made of the same metal. The small cone is 6 cm tall and has mass 540 g. The large cone has mass 4.32 kg.
   (a) Calculate the height of the large cone. [2]
   (b) The small cone has surface area 90 cm^2. Calculate the surface area of the large cone. [2]

Challenge

(E) A cone is 12 cm tall and has volume 320 cm^3. A plane parallel to the base cuts off the top 4 cm, leaving a small cone on top and a frustum below.

(a) Explain why the small cone is similar to the original cone, and write down the length scale factor.

(b) Find the volume of the small cone.

(c) Find the volume of the frustum, correct to 3 significant figures. (Frustums return in Ep 80.)

Answers

Set A

1. (a) 1 : 9 (b) 1 : 27

2. k = 4, $k^2 = 16$: 15 × 16 = 240 cm^2

3. Area factor 45/20 = 9/4, so k = 3/2: 6 × 1.5 = 9 cm

4. 1 cm represents 20 000 cm = 0.2 km, so 1 cm^2 represents 0.04 km^2: 4.5 × 0.04 = 0.18 km^2

5. k = 1.5, $k^3 = 3.375$: 160 × 3.375 = 540 ml

6. Volume factor 125/27, k = 5/3: 10 ÷ 5/3 = 6 cm

7. Lengths 2 : 3, so volumes 8 : 27

8. Mass factor 8, so k = 2: 60 cm

Set B

1. (a) k = 12/8 = 3/2; area factor 9/4 M1; 360 × 4/9 = 160 cm^2 A1.
   (b) Volume factor 27/8 M1; 96 × 27/8 = 324 cm^3 A1.

2. Volume factor 500/330 M1; k = $\sqrt[3]{(500/330)}$ = 1.1486… M1; height = 14 × 1.1486… = 16.08… ≈ 16.1 cm A1.

3. (a) Angle A is common B1; angle ADE = angle ABC (corresponding angles, DE ∥ BC) B1.
   (b) AD : AB = 2 : 5, area factor 25/4 M1; area ABC = 12 × 25/4 = 75 cm^2 A1.
   (c) 75 − 12 = 63 cm^2 B1.

4. (a) Mass factor 4320/540 = 8, so k = $\sqrt[3]{8}$ = 2 M1; height 12 cm A1.
   (b) 90 × 2^2 M1 = 360 cm^2 A1.

Challenge

(a) The cut is parallel to the base, so the small cone has the same angles (same shape); it is 4 cm tall against 12 cm, so k = 1/3.
(b) 320 × $(1/3)^3$ = 320/27 = 11.85… ≈ 11.9 cm^3.
(c) 320 − 320/27 = 8320/27 = 308.1… ≈ 308 cm^3.