IM-059 · Differentiation: the gradient function

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 059

Extended only (E2.12.2). If $y = ax^n$, then $dy/dx = nax^{n-1}$; a constant differentiates to 0. Differentiate sums term by term, expand brackets first, and find the gradient at a point by substituting into dy/dx (not into y).

Set A · Fluency

1. (E) Find dy/dx for (a) $y = x^5$ (b) $y = 4x^3$ (c) $y = -3x$ (d) $y = 12$.

2. (E) Find dy/dx for $y = 3x^4 - 2x^2 + 8$.

3. (E) Find dy/dx for $y = \frac{1}{2}x^2 + 6x - 1$.

4. (E) Find dy/dx for $y = \frac{1}{4}x^4 - x^3$.

5. (E) Find the gradient of $y = x^2 - 5x$ at $x = 4$.

6. (E) Find the gradient of $y = (x - 3)^2$ at $x = 5$.

7. (E) For $y = x(4 - x)$, find dy/dx and the gradient at $x = 3$.

8. (E) For $y = (2x + 1)(x - 3)$, find dy/dx and the gradient where the curve crosses the y-axis.

Set B · Exam style

1. (Paper 1/2 · no calculator) (E) A curve has equation $y = x^3 - 4x^2 + 3$.
   (a) Find dy/dx. [2]
   (b) Show that (2, −5) lies on the curve, and find the gradient there. [2]
   (c) Find the values of x where the gradient is zero. [2]

2. (Paper 1/2 · no calculator) (E) $y = (3x - 2)^2$.
   (a) Expand the bracket and find dy/dx. [3]
   (b) Find the gradient of the curve where it crosses the y-axis. [1]

3. (Paper 1/2 · no calculator) (E) A ball is thrown upwards. Its height is y metres after x seconds, where $y = 30x - 5x^2$.
   (a) Find dy/dx. [1]
   (b) Find the rate at which the height is changing when $x = 1$, with units. [1]
   (c) Find dy/dx when $x = 4$ and explain what the sign tells you. [2]

4. (Paper 1/2 · no calculator) (E) The curve $y = ax^2 + bx$ has gradient 7 at $x = 1$ and gradient 11 at $x = 2$. Find a and b. [4]

Challenge

(E, uses tangents and chords, Ep 58) In Ep 58 a tangent drawn by eye to $y = x^2 - 4x + 5$ at (3, 2) gave a gradient of about 2.
(a) Use differentiation to find the exact gradient at (3, 2).
(b) Show that the chord from (3, 2) to the point on the curve with $x = 3 + h$ has gradient $2 + h$, and explain how this confirms your answer to (a).

Spaced review

S1. (Ep 50, Paper 1/2) A is (1, 2) and B is (7, 10). Find the midpoint of AB and the length of AB.

S2. (Ep 46, Paper 1/2) $f(x) = 3x - 2$. Find $f(4)$, and solve $f(x) = 13$.

Answers

Set A

1. (a) $5x^4$ (b) $12x^2$ (c) -3 (d) 0.

2. $12x^3 - 4x$.

3. $x + 6$.

4. $x^3 - 3x^2$.

5. $2x - 5$ at $x = 4$: 3.

6. $x^2 - 6x + 9$, $dy/dx = 2x - 6$; at $x = 5$: 4.

7. $4x - x^2$, $dy/dx = 4 - 2x$; at $x = 3$: -2.

8. $2x^2 - 5x - 3$, $dy/dx = 4x - 5$; at $x = 0$: -5.

Set B

1. (a) $3x^2 - 8x$ B1 B1. (b) $8 - 16 + 3 = -5$ B1; $12 - 16 = -4$ B1. (c) $x(3x - 8) = 0$ M1; $x = 0$ or $x = 8/3$ A1.

2. (a) $9x^2 - 12x + 4$ M1 A1; $dy/dx = 18x - 12$ B1 (FT). (b) $x = 0$: -12 B1.

3. (a) $30 - 10x$ B1. (b) 20 m/s B1. (c) -10 B1; negative, so the height is decreasing: the ball is coming down (at 10 m/s) B1.

4. $dy/dx = 2ax + b$ M1; $2a + b = 7$ and $4a + b = 11$ M1; $a = 2$ A1; $b = 3$ A1.

Challenge

(a) $dy/dx = 2x - 4$; at $x = 3$ the gradient is exactly 2.
(b) $((3 + h)^2 - 4(3 + h) + 5 - 2) \div h = (2h + h^2) \div h = 2 + h$. As h gets closer to 0, the chord gradient approaches 2, the gradient at (3, 2).

Spaced review

S1. Midpoint (4, 6); length $\sqrt{(6^2 + 8^2)} = \sqrt{100} = 10$.

S2. $f(4) = 10$; $3x - 2 = 13$ gives $x = 5$.