IM-058 · Gradients of curves: drawing tangents

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 058

Extended only (E2.9.1, E2.12.1). A tangent touches the curve at the point without crossing it there; draw it long and ruled, and measure its gradient with a large triangle. Give units for rates. Answers from hand-drawn tangents are estimates; a sensible range is given.

Set A · Fluency

1. (E) On the curve $y = x^2$, is the gradient positive, negative or zero at (a) $x = -3$ (b) $x = 0$ (c) $x = 2$?

2. (E) What is the gradient of the tangent at the highest point of an arch-shaped curve? Explain.

3. (E) The tangent to a curve at P passes through (1, 2) and (5, 14). Find the gradient of the curve at P.

4. (E) The tangent to a curve at Q passes through (0, 9) and (6, 0). Find the gradient of the curve at Q.

5. (E) On a distance–time graph (distance in metres, time in seconds) the tangent at $t = 10$ passes through (4, 0) and (16, 30). Estimate the speed at $t = 10$, with units.

6. (E) For $y = x^2$, find the gradient of the chord from (3, 9) to (3.1, 9.61), and from (3, 9) to (3.01, 9.0601). What do you think the gradient of the curve at (3, 9) is?

7. (E) Complete a table for $y = x^2 - 2x + 3$ for $-1 \le x \le 3$, draw the graph, and draw the tangent at $x = 2$. Estimate the gradient there.

Set B · Exam style

1. (Paper 3/4 · calculator) (E)
   (a) Complete a table of values for $y = 10/x$ for $x = 1, 2, \ldots, 10$. Give values to 2 decimal places. [2]
   (b) Draw the graph of $y = 10/x$ for $1 \le x \le 10$. [3]
   (c) By drawing a tangent, estimate the gradient of the curve at $x = 2$. [3]

2. (Paper 3/4 · calculator) (E) The number of fish in a lake after t months is $P = 200 \times 1.5^t$.
   (a) Complete a table of values for $t = 0, 1, 2, 3, 4$. [2]
   (b) Draw the graph of P against t. [3]
   (c) By drawing a tangent, estimate the rate at which the number of fish is increasing when $t = 2$. Give the units. [3]

3. (Paper 1/2 · no calculator) (E) A and B are points on the curve $y = x^2$. A is (2, 4) and B has x-coordinate $2 + h$.
   (a) Show that the gradient of the chord AB is $4 + h$. [3]
   (b) Hence write down the gradient of the curve at A. [1]

Challenge

(E, uses distance–time graphs, Ep 52) A car moves so that its distance d metres after t seconds is $d = 30t - t^2$, for $0 \le t \le 15$.

(a) Complete a table for t = 0, 3, 6, 9, 12, 15 and draw the graph.

(b) Work out the average speed over the 15 seconds.

(c) By drawing a tangent, estimate the speed at t = 5.

(d) Draw a tangent that is parallel to the chord from (0, 0) to (15, 225). At what time is the car's speed equal to its average speed?

Answers

Set A

1. (a) negative (b) zero (c) positive.

2. Zero: the tangent at the top of an arch is horizontal.

3. 12 ÷ 4 = 3.

4. −9 ÷ 6 = −1.5.

5. 30 ÷ 12 = 2.5 m/s.

6. 6.1 and 6.01; the gradient at (3, 9) is 6.

7. y: 6, 3, 2, 3, 6. The gradient at x = 2 is about 2 (accept 1.6 to 2.4).

Set B

1. (a) 10, 5, 3.33, 2.5, 2, 1.67, 1.43, 1.25, 1.11, 1 B2. (b) points plotted B2 FT, smooth curve B1. (c) tangent at (2, 5) B1; rise ÷ run from it M1; about −2.5 A1 (accept −2 to −3).

2. (a) 200, 300, 450, 675, 1012.5 B2. (b) points plotted B2 FT, smooth curve B1. (c) tangent at (2, 450) B1; rise ÷ run M1; about 180 fish per month A1 (true value 182.5; accept 160 to 200 with units).

3. (a) B is $(2 + h, (2 + h)^2)$ B1; gradient $((2 + h)^2 − 4) ÷ h = (4h + h^2) ÷ h$ M1; $= 4 + h$ A1. (b) As h approaches 0, the gradient approaches 4 B1.

Challenge

(a) d: 0, 81, 144, 189, 216, 225. (b) 225 ÷ 15 = 15 m/s. (c) about 20 m/s (accept 18 to 22). (d) The chord has gradient 15; the parallel tangent touches at t = 7.5 s, when d = 168.75.