IM-056 · Exponential graphs and sketching curves

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 056

Extended only (E2.10, E2.11). For $y = a \times b^x + c$: the y-intercept is $a + c$ and the asymptote is $y = c$. A sketch shows the shape and ends, the intercepts, turning points, symmetry and asymptotes (broken and labelled).

Set A · Fluency

1. (E) Work out $y = 2^x + 3$ for $x = -2, -1, 0, 1, 2, 3$.

2. (E) Write down the y-intercept and the equation of the asymptote of $y = 2^x + 3$.

3. (E) Write down the y-intercept and the equation of the asymptote of $y = 6 \times 3^x - 2$.

4. (E) For each formula, say whether it shows growth or decay, and give the percentage change per year.
   (a) $V = 500 \times 0.9^t$ (b) $N = 200 \times 1.15^t$

5. (E) Factorise $x^3 - 9x$ and write down the roots of $y = x^3 - 9x$.

6. (E) Write down the root and the y-intercept of $y = x^3 - 27$.

7. (E) Write down the asymptotes of $y = 6/x - 2$ and find where the graph crosses the x-axis.

8. (E) Name the family of each graph (linear, quadratic, cubic, reciprocal or exponential).
   (a) $y = 5 - 2x$ (b) $y = 3/x$ (c) $y = 2x^2 - 1$ (d) $y = 0.5^x$ (e) $y = x^3 + 1$

Set B · Exam style

1. (Paper 3/4 · calculator) (E)
   (a) Complete a table of values for $y = 3 \times 2^x - 4$ for $x = -2, -1, 0, 1, 2$. [2]
   (b) Draw the graph for $-2 \le x \le 2$. [3]
   (c) Use your graph to solve $3 \times 2^x - 4 = 0$. [1]
   (d) Write down the equation of the asymptote of $y = 3 \times 2^x - 4$. [1]

2. (Paper 3/4 · calculator) (E) A car costs 15 000 dollars. Its value falls by 18% each year.
   (a) Write down a formula for its value V dollars after t years. [1]
   (b) Work out its value after 3 years. [2]
   (c) Draw the graph of V for $0 \le t \le 5$ and use it to estimate when the car is worth 7500 dollars. [3]

3. (Paper 1/2 · no calculator) (E) Sketch $y = x^3 - x^2 - 6x$, showing the coordinates of the points where it meets the axes. [3]

4. (Paper 1/2 · no calculator) (E) The graph of $y = a \times 3^x + c$ passes through (0, 5) and (1, 13).
   (a) Find a and c. [3]
   (b) Write down the equation of the asymptote. [1]
   (c) Find y when $x = -1$. [1]

Challenge

(E, uses growth and decay, Ep 19) Town A has 1000 people and grows by 20% a year: $A = 1000 \times 1.2^t$. Town B has 4000 people and shrinks by 20% a year: $B = 4000 \times 0.8^t$.

(a) Sketch both graphs on the same axes, labelling their intercepts and the asymptote.

(b) Work out A and B for t = 0, 1, 2, 3, 4. After how many whole years is A first larger than B?

(c) Explain why the model says town B never reaches 0 people.

Spaced review

S1. (Ep 7, Paper 1/2) Work out (a) 16^(−½) (b) 8^(⅔).

S2. (Ep 30, Paper 1/2) Solve the simultaneous equations 3x + 2y = 16 and x − 2y = 0.

Answers

Set A

1. 3.25, 3.5, 4, 5, 7, 11.

2. y-intercept 4; asymptote $y = 3$.

3. y-intercept $6 - 2 = 4$; asymptote $y = -2$.

4. (a) decay, 10% decrease (b) growth, 15% increase.

5. $x(x - 3)(x + 3)$; roots -3, 0, 3.

6. Root $x = 3$ ($\sqrt[3]{27} = 3$); y-intercept -27.

7. $x = 0$ and $y = -2$; $6/x = 2$ gives $x = 3$: (3, 0).

8. (a) linear (b) reciprocal (c) quadratic (d) exponential (e) cubic.

Set B

1. (a) -3.25, -2.5, -1, 2, 8 B2 (B1 for 3 or 4 correct). (b) points plotted B2 FT, smooth curve above $y = -4$ B1. (c) $x \approx 0.4$ B1 (accept 0.3 to 0.5). (d) $y = -4$ B1.

2. (a) $V = 15\,000 \times 0.82^t$ B1. (b) $15\,000 \times 0.82^3$ M1; 8270.52 dollars A1. (c) table 15 000, 12 300, 10 086, 8271, 6782, 5561 and smooth decay curve B2; $t \approx 3.5$ years B1 (accept 3.4 to 3.6).

3. $x(x - 3)(x + 2)$ M1; roots $(-2, 0)$, $(0, 0)$, $(3, 0)$ A1; cubic shape bottom left to top right through these points B1.

4. (a) $a + c = 5$ and $3a + c = 13$ M1; $a = 4$ A1; $c = 1$ A1. (b) $y = 1$ B1. (c) $4 \times \frac{1}{3} + 1 = 7/3$ B1.

Challenge

(a) A: rising from (0, 1000); B: falling from (0, 4000); both have the asymptote $y = 0$ (B on the right, A on the left).
(b) $t = 0$: 1000, 4000; $t = 1$: 1200, 3200; $t = 2$: 1440, 2560; $t = 3$: 1728, 2048; $t = 4$: 2073.6, 1638.4. After 4 years.
(c) Each year B is multiplied by 0.8; 0.8 times a positive number is still positive, so B gets closer to 0 but never reaches it.

Spaced review

S1. (a) $1/\sqrt{16} = 1/4$ (b) $(\sqrt[3]{8})^2 = 4$.

S2. Add: $4x = 16$, $x = 4$; $y = 2$.