IM-051 · Parallel and perpendicular lines

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 051

Core + Extended (C3.6, E3.6, E3.7). Questions tagged (E) are Extended only. Give equations fully simplified.

Set A · Fluency

1. Write down the gradient of a line parallel to
   (a) $y = 5x - 2$ (b) $y = 3 - x$ (c) $2y = 7x + 4$

2. Find the equation of the line parallel to $y = 4x + 1$ that passes through $(1, 9)$.

3. Find the equation of the line parallel to $y = -3x$ that passes through $(2, -4)$.

4. Which of these lines are parallel to $y = 2x$? Show your working.
   (a) $y = 2x + 7$ (b) $4x - 2y = 3$ (c) $y = 7 - 2x$ (d) $6x = 3y + 1$

5. (E) Write down the gradient of a line perpendicular to $y = -\frac{3}{4}x + 2$.

6. (E) Find the gradient of a line perpendicular to (a) $y = 6x$ (b) $5x + 2y = 1$.

7. (E) Show that the lines $y = 3x + 2$ and $x + 3y = 9$ are perpendicular.

8. (E) Find the equation of the line through $(3, -2)$ that is perpendicular to $y = \frac{1}{2}x + 4$.

Set B · Exam style

1. (Paper 1/2 · no calculator) (C) The line L has equation $y = 3x - 4$.
   (a) Write down the gradient of a line parallel to L. [1]
   (b) Find the equation of the line parallel to L that passes through $(-2, 5)$. [2]

2. (Paper 1/2 · no calculator) (E) A is the point $(-4, 1)$ and B is the point $(2, 5)$.
   Find the equation of the perpendicular bisector of AB. Give your answer in the form $ax + by = c$, where a, b and c are integers. [5]

3. (Paper 1/2 · no calculator) (E) P is $(1, 2)$, Q is $(7, 5)$ and R is $(5, 9)$.
   (a) Show that angle PQR is a right angle. [3]
   (b) Find the area of triangle PQR. [2]

Challenge

(E, uses Ep 30 and Ep 50) P is the point $(7, -1)$ and L is the line $y = x + 2$.
(a) Find the equation of the line through P perpendicular to L.
(b) Find the coordinates of the point F where this line meets L.
(c) Find the shortest distance from P to L, as a simplified surd.

Answers

Set A

1. (a) 5 (b) −1 (c) 7/2.

2. 9 = 4(1) + c, c = 5: y = 4x + 5.

3. −4 = −3(2) + c, c = 2: y = −3x + 2.

4. (a) m = 2 ✓ (b) 2y = 4x − 3, y = 2x − 1.5, m = 2 ✓ (c) m = −2 ✗ (d) y = 2x − ⅓, m = 2 ✓. Parallel: (a), (b), (d).

5. 4/3.

6. (a) −1/6 (b) y = −(5/2)x + ½, m = −5/2, so the perpendicular gradient is 2/5.

7. x + 3y = 9 gives y = −⅓x + 3; 3 × (−⅓) = −1, so the lines are perpendicular.

8. Perpendicular gradient −2; −2 = −2(3) + c, c = 4: y = −2x + 4.

Set B

1. (a) 3 B1 (b) 5 = 3(−2) + c M1; y = 3x + 11 A1.

2. Midpoint (−1, 3) B1; gradient AB = 4/6 = 2/3 B1; perpendicular gradient −3/2 M1; 3 = −(3/2)(−1) + c, c = 3/2 M1; 3x + 2y = 3 A1.

3. (a) m(PQ) = 3/6 = ½ B1; m(QR) = 4/(−2) = −2 B1; ½ × (−2) = −1, so PQ ⊥ QR and angle PQR = 90° B1.
   (b) PQ = √45 = 3√5, QR = √20 = 2√5 M1; area = ½ × 3√5 × 2√5 = 15 A1.

Challenge

(a) Gradient of L is 1, so the perpendicular gradient is −1: −1 = −7 + c, c = 6: y = −x + 6.
(b) x + 2 = −x + 6, x = 2, y = 4: F(2, 4).
(c) PF = √((7 − 2)² + (−1 − 4)²) = √50 = 5√2.