IM-046 · Functions I: notation, domain and range

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 046

Extended only (E2.13.1). f(a) = the output when the input is a. Describe a domain with x and a range with f(x).

Set A · Fluency

1. (E) $f(x) = 3x + 2$. Find $f(4)$, $f(-5)$ and $f(0.5)$.

2. (E) $f(x) = 6 - 4x$. Find $f(-3)$ and $f(½)$.

3. (E) $g(x) = x^2 - 3x$. Find $g(-2)$, and simplify $g(2a)$.

4. (E) $g(x) = 3(x - 2)/4$. Solve $g(x) = 6$.

5. (E) $h(x) = 2x^2 - 5$. Solve $h(x) = 45$.

6. (E) Write down the value of x that must be excluded from the domain of
   (a) $4/(x - 7)$ (b) $1/(3x + 6)$.
   (c) Write down the domain of $\sqrt{(x + 4)}$.

7. (E) Find the range of
   (a) $f(x) = x^2 + 6$, for all real x (b) $g(x) = 4x - 1$, with domain $1 \le x \le 5$.

8. (E) $f : x \mapsto x^2 - 1$ has domain $\{-2, -1, 0, 1, 2, 3\}$. Draw a mapping diagram and list the range. Which outputs have two arrows arriving?

Set B · Exam style

1. (Paper 1/2 · no calculator) (E) $f(x) = 2x - 9$ and $g(x) = (x + 5)/3$.
   (a) Find $f(-4)$. [1]
   (b) Solve $f(x) = 7$. [2]
   (c) Solve $f(x) = g(x)$. [2]

2. (Paper 1/2 · no calculator) (E) $h(x) = 1 + 8/(x - 3)$.
   (a) Find $h(5)$. [1]
   (b) Write down the value of x that is not in the domain of h. [1]
   (c) Solve $h(x) = 3$. [2]

3. (Paper 3/4 · calculator) (E) $q(x) = 0.5x^2 - 3.2$, with domain $-2 \le x \le 4$.
   (a) Find the range of q. [2]
   (b) Solve $q(x) = 2$ for x in the domain. Give your answer correct to 3 significant figures. [2]

Challenge

(E, uses completing the square, Ep 33) $f(x) = x^2 - 8x + 21$.
(a) Write f(x) in the form $(x - p)^2 + q$.
(b) Find the range of f when the domain is all real numbers.
(c) Find the range of f when the domain is $0 \le x \le 3$.

Answers

Set A

1. 14, −13, 3.5.

2. 18, 4.

3. g(−2) = 4 + 6 = 10; g(2a) = 4a² − 6a.

4. 3(x − 2) = 24, x − 2 = 8, x = 10.

5. 2x² = 50, x² = 25, x = 5 or x = −5.

6. (a) x = 7 (b) x = −2 (c) x ≥ −4.

7. (a) f(x) ≥ 6 (b) 3 ≤ g(x) ≤ 19.

8. f(−2) = 3, f(−1) = 0, f(0) = −1, f(1) = 0, f(2) = 3, f(3) = 8. Range {−1, 0, 3, 8}. The outputs 0 and 3 each have two arrows arriving.

Set B

1. (a) −17 B1. (b) 2x − 9 = 7 M1, x = 8 A1. (c) 2x − 9 = (x + 5)/3, so 6x − 27 = x + 5 M1; 5x = 32, x = 6.4 A1.

2. (a) 1 + 8/2 = 5 B1. (b) x = 3 B1. (c) 8/(x − 3) = 2 M1, x − 3 = 4, x = 7 A1.

3. (a) q(−2) = −1.2, q(4) = 4.8, minimum q(0) = −3.2 M1; −3.2 ≤ q(x) ≤ 4.8 A1.
   (b) 0.5x² = 5.2, x² = 10.4 M1; x = 3.22 (x = −3.22 is outside the domain) A1.

Challenge

(a) (x − 4)² + 5.
(b) f(x) ≥ 5.
(c) The vertex is at x = 4, outside 0 ≤ x ≤ 3, so f decreases across the domain: f(0) = 21, f(3) = 6. Range 6 ≤ f(x) ≤ 21.