IM-044 · Sequences II: quadratic, cubic and exponential

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 044

Core: simple quadratic ($n^2 + k$) and simple cubic ($n^3 + k$). Extended: general quadratic, exponential and combinations (tagged (E)).

Set A · Fluency

1. Write down the next two terms of each sequence.
   (a) 1, 8, 27, 64, … (b) 1, 3, 6, 10, 15, … (c) 3, 6, 11, 18, 27, …

2. Find the nth term of 0, 7, 26, 63, 124, …

3. Find the nth term of 10, 13, 18, 25, 34, …

4. The nth term of a sequence is $n^2 - 2$. (a) Is 98 a term? (b) Is 60 a term? Explain how you decide.

5. (E) Find the nth term of 5, 9, 15, 23, 33, …

6. (E) Find the nth term of 1, 10, 25, 46, 73, …

7. (E) Find the nth term of (a) 7, 14, 28, 56, … (b) 162, 54, 18, 6, …

8. (E) Find the nth term of 1/3, 2/5, 3/7, 4/9, …

Set B · Exam style

1. (Paper 1/2 · no calculator) (C) Each pattern in a sequence is an n by n square of dots with one extra dot at each corner, outside the square. Pattern 1 has 5 dots, pattern 2 has 8, pattern 3 has 13, pattern 4 has 20.
   (a) How many dots are in pattern 5? [1]
   (b) Find an expression for the number of dots in pattern n. [2]
   (c) Which pattern has exactly 404 dots? [1]

2. (Paper 1/2 · no calculator) (E) Here are the first five terms of a sequence: 2, 9, 20, 35, 54, …
   (a) Find an expression for the nth term. [3]
   (b) Find the 20th term. [1]
   (c) Which term is equal to 252? [2]

3. (Paper 3/4 · calculator) (E) A sequence begins 12, 15, 18.75, …, each term being the same multiple of the one before.
   (a) Find an expression for the nth term. [2]
   (b) Find the first term of the sequence that is greater than 100, and its position. [2]

Challenge

(E, uses Ep 31) Sequence T has nth term $n^2 + 3n$. Sequence U has nth term $7n + 12$.
(a) For which value of n is $T_n = U_n$? What is the common value?
(b) Show that every term of T is an even number.

Answers

Set A

1. (a) 125, 216 (b) 21, 28 (c) 38, 51 (nth term $n^2 + 2$).

2. $n^3 - 1$.

3. $n^2 + 9$.

4. (a) $n^2 = 100$, n = 10: yes, the 10th term. (b) $n^2 = 62$, which is not a square number: no.

5. Second difference 2, a = 1; term − n^2 = 4, 5, 6, 7, 8 = n + 3. nth term $n^2 + n + 3$.

6. Second difference 6, a = 3; term − $3n^2$ = −2 each time. nth term $3n^2 - 2$.

7. (a) $7 \times 2^{n-1}$ (b) $162 \times (1/3)^{n-1}$.

8. $n/(2n + 1)$.

Set B

1. (a) 29 B1. (b) $n^2 + 4$ B2 (B1 for $n^2 + k$). (c) $n^2 = 400$, pattern 20 B1.

2. (a) Second difference 4, so $2n^2$ M1; term − $2n^2$ = 0, 1, 2, 3, 4 M1; $2n^2 + n - 1$ A1.
   (b) 800 + 20 − 1 = 819 B1.
   (c) $2n^2 + n - 253 = 0$, $(2n + 23)(n - 11) = 0$ M1; n = 11 (the 11th term) A1.

3. (a) r = 15 ÷ 12 = 1.25 B1; $12 \times 1.25^{n-1}$ B1.
   (b) Trial: $T_{10} = 12 \times 1.25^9 = 89.4\ldots$, $T_{11} = 12 \times 1.25^{10} = 111.75\ldots$ M1; the first term greater than 100 is the 11th, 112 (3 s.f.) A1.

Challenge

(a) $n^2 + 3n = 7n + 12$, so $n^2 - 4n - 12 = 0$, $(n - 6)(n + 2) = 0$, n = 6 (n = −2 is not a position). The common value is 54.
(b) $n^2 + 3n = n(n + 3)$. If n is even, n(n + 3) is even. If n is odd, n + 3 is even, so n(n + 3) is even. So every term is even.