IM-039 · Changing the subject II: powers, roots, and the subject twice

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 039

All of this sheet is Extended (E2.5.7). Core students: IM-038 covers the Core part.

Set A · Fluency

1. (E) The surface area of a sphere is $A = 4\pi r^2$. Make r the subject.

2. (E) Make x the subject of $y = 5x^2 - 3$.

3. (E) Make q the subject of $p = \sqrt{(q/2)}$.

4. (E) Make m the subject of $T = 3\sqrt{(m + 1)}$.

5. (E) Make x the subject of $ax - b = cx + k$.

6. (E) Make x the subject of $y(x + 2) = 3x$.

7. (E) Make x the subject of $y = (x + 4)/(x - 1)$.

8. (E) The volume of a cylinder is $V = \pi r^2 h$. Make r the subject.

Set B · Exam style

1. (Paper 2 · no calculator) (E) Make x the subject of $2x + a = b(x - 3)$. [3]

2. (Paper 2 · no calculator) (E) Make y the subject of $x = (3y + 2)/(y - 4)$. [4]

3. (Paper 4 · calculator) (E) The time, T seconds, for one swing of a pendulum of length l metres is $T = 2\pi\sqrt{(l/9.81)}$.
   (a) Make l the subject of the formula. [3]
   (b) Find the length of a pendulum with T = 1.5. Give your answer correct to 3 significant figures. [1]

4. (Paper 2 · no calculator) (E) A ring has outer radius R and inner radius r. Its area is $A = \pi(R^2 - r^2)$. Make r the subject of the formula. [3]

Challenge

(E) (a) Make x the subject of $y = (2x^2 + 1)/(x^2 - 3)$.
(b) Find the exact values of x when y = 4. Give your answer with a rational denominator (Ep 21).

Answers

Set A

1. $r = \sqrt{(A/(4\pi))}$ (positive: a radius)

2. $x = \pm\sqrt{((y + 3)/5)}$

3. $q = 2p^2$

4. $T/3 = \sqrt{(m + 1)}$, $m = T^2/9 - 1$

5. $x(a - c) = b + k$, $x = (b + k)/(a - c)$

6. $xy + 2y = 3x$, $x(y - 3) = -2y$, $x = 2y/(3 - y)$

7. $xy - y = x + 4$, $x(y - 1) = y + 4$, $x = (y + 4)/(y - 1)$

8. $r = \sqrt{(V/(\pi h))}$

Set B

1. $2x + a = bx - 3b$ M1; $2x - bx = -3b - a$, $x(2 - b) = -(a + 3b)$ M1; $x = (a + 3b)/(b - 2)$ A1 (or $-(a + 3b)/(2 - b)$).

2. $xy - 4x = 3y + 2$ M1; $xy - 3y = 4x + 2$ M1; $y(x - 3) = 4x + 2$ M1; $y = (4x + 2)/(x - 3)$ A1.

3. (a) $T/(2\pi) = \sqrt{(l/9.81)}$ M1; $T^2/(4\pi^2) = l/9.81$ M1; $l = 9.81T^2/(4\pi^2)$ A1. (b) $9.81 \times 2.25/(4\pi^2) = 0.5591\ldots = 0.559$ m B1.

4. $A/\pi = R^2 - r^2$ M1; $r^2 = R^2 - A/\pi$ M1; $r = \sqrt{(R^2 - A/\pi)}$ A1 (positive: a radius). Note $\sqrt{(R^2 - A/\pi)}$ is not $R - \sqrt{(A/\pi)}$.

Challenge

(a) $y(x^2 - 3) = 2x^2 + 1$, $yx^2 - 2x^2 = 3y + 1$, $x^2(y - 2) = 3y + 1$, $x = \pm\sqrt{((3y + 1)/(y - 2))}$.
(b) $x^2 = 13/2$, $x = \pm\sqrt{(13/2)} = \pm\sqrt{26}/2$. Check: $(2 \times 13/2 + 1)/(13/2 - 3) = 14/(7/2) = 4$.