IM-035 · Problems that lead to quadratic equations

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 035

Tier: Extended only (E). In every question, reject any impossible solution and say why.

Set A · Fluency

1. (E) A rectangle is x cm wide and (x + 5) cm long. Its area is 84 cm². Find its length and width.

2. (E) The side of a square is increased by 3 cm. The new square has area 64 cm². Find the side of the original square.

3. (E) The two shorter sides of a right-angled triangle are x cm and (x + 2) cm. The hypotenuse is 10 cm. Find x.

4. (E) The product of two consecutive positive integers is 72. Find the integers.

5. (E) The sum of the squares of two consecutive integers is 85. Find the two possible pairs of integers.

6. (E) A rectangle has perimeter 26 cm and area 40 cm². Find the lengths of its sides.

7. (E) A triangle has height x cm and base (x + 4) cm. Its area is 30 cm². Find the height and the base.

8. (E) A walker covers 30 km at v km/h. If she walked 1 km/h faster, she would take 1 hour less. Find v.

Set B · Exam style

1. (Paper 1/2 · no calculator) (E) A rectangular lawn is (2x + 1) m long and (x + 3) m wide. Its area is 63 m².
   (a) Show that $2x^2 + 7x - 60 = 0$. [2]
   (b) Find the length and width of the lawn. [3]

2. (Paper 3/4 · calculator) (E) The hypotenuse of a right-angled triangle is 15 cm. One of the other sides is 4 cm longer than the third side. Find the length of the shortest side, correct to 3 significant figures. [4]

3. (Paper 1/2 · no calculator) (E) When a positive number is added to its square, the result is 110. Find the number. [3]

4. (Paper 3/4 · calculator) (E) A car travels 150 km at v km/h. If its speed were 10 km/h faster, the journey would take 30 minutes less.
   (a) Show that $v^2 + 10v - 3000 = 0$. [3]
   (b) Find v. [2]

Challenge

(E) A farmer has 40 m of fencing to make three sides of a rectangular pen. The fourth side is a wall. The two sides at right angles to the wall are each x m long.
(a) Show that the area of the pen is $A = x(40 - 2x)$ m².
(b) By completing the square (Ep 33), find the largest possible area and the value of x that gives it.
(c) Find the two values of x that give an area of 150 m².

Spaced review

R1. (Ep 16) Increase 240 by 15%.

R2. (Ep 13) Share 84 in the ratio 3 : 4.

Answers

Set A

1. $x(x + 5) = 84$, $x^2 + 5x - 84 = 0$, $(x + 12)(x - 7) = 0$: $x = 7$ (-12 rejected). Width 7 cm, length 12 cm.

2. $(x + 3)^2 = 64$, $x + 3 = 8$ (-8 rejected), $x = 5$ cm.

3. $x^2 + (x + 2)^2 = 100$, $x^2 + 2x - 48 = 0$, $(x + 8)(x - 6) = 0$: $x = 6$ (-8 rejected). Sides 6, 8, 10 cm.

4. $n(n + 1) = 72$, $n^2 + n - 72 = 0$, $(n + 9)(n - 8) = 0$: 8 and 9 (-9 rejected: not positive).

5. $n^2 + (n + 1)^2 = 85$, $n^2 + n - 42 = 0$, $(n + 7)(n - 6) = 0$: 6 and 7, or -7 and -6.

6. $l + w = 13$, $w(13 - w) = 40$, $w^2 - 13w + 40 = 0$: sides 5 cm and 8 cm.

7. $\frac{1}{2}x(x + 4) = 30$, $x^2 + 4x - 60 = 0$, $(x + 10)(x - 6) = 0$: height 6 cm, base 10 cm.

8. $30/v - 30/(v + 1) = 1$, $v^2 + v - 30 = 0$, $(v + 6)(v - 5) = 0$: $v = 5$ km/h (-6 rejected).

Set B

1. (a) $(2x + 1)(x + 3) = 63$ M1; $2x^2 + 7x + 3 = 63$, so $2x^2 + 7x - 60 = 0$ A1.
   (b) $(2x + 15)(x - 4) = 0$ M1; $x = 4$ ($x = -15/2$ rejected, the width would be negative) A1; length 9 m, width 7 m A1.

2. $x^2 + (x + 4)^2 = 225$ M1; $2x^2 + 8x - 209 = 0$ A1; $x = (-8 + \sqrt{1736})/4$ M1; $x = 8.42$ cm (the negative root is rejected) A1.

3. $n^2 + n - 110 = 0$ M1; $(n + 11)(n - 10) = 0$ M1; $n = 10$ (-11 rejected: not positive) A1.

4. (a) $150/v - 150/(v + 10) = 1/2$ M1; $300(v + 10) - 300v = v(v + 10)$ M1; $3000 = v^2 + 10v$, so $v^2 + 10v - 3000 = 0$ A1.
   (b) $(v + 60)(v - 50) = 0$ M1; $v = 50$ km/h (-60 rejected) A1.

Challenge

(a) The two sides at right angles to the wall use $2x$ m, so the side opposite the wall is $(40 - 2x)$ m, and $A = x(40 - 2x)$.
(b) $A = 40x - 2x^2 = -2(x^2 - 20x) = -2[(x - 10)^2 - 100] = 200 - 2(x - 10)^2$. Largest area 200 m², when $x = 10$.
(c) $x(40 - 2x) = 150$, $2x^2 - 40x + 150 = 0$, $x^2 - 20x + 75 = 0$, $(x - 5)(x - 15) = 0$: $x = 5$ or $x = 15$ (both possible: 5 m by 30 m, or 15 m by 10 m).

Spaced review

R1. $240 \times 1.15 = 276$.

R2. $84 \div 7 = 12$: 36 and 48.