IM-034 · Simultaneous equations: one linear, one non-linear

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 034

Tier: Extended only (E).

Set A · Fluency

1. (E) Solve the simultaneous equations $y = x + 2$ and $y = x^2$.

2. (E) Solve the simultaneous equations $y = 3x - 2$ and $y = x^2$.

3. (E) Solve the simultaneous equations $y = 2x + 3$ and $y = x^2 + 2x - 1$.

4. (E) Solve the simultaneous equations $x + y = 5$ and $xy = 6$.

5. (E) Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$.

6. (E) Solve the simultaneous equations $y = 2x - 1$ and $x^2 + y^2 = 2$.

7. (E) Show that the line $y = 2x - 1$ is a tangent to the curve $y = x^2$, and find the point of contact.

8. (E) Show that the line $y = x - 3$ does not meet the curve $y = x^2 + x$.

Set B · Exam style

1. (Paper 1/2 · no calculator) (E) Solve the simultaneous equations. Show all your working.
   $y = x^2 - 4x + 1$
   $y = 2x - 7$ [4]

2. (Paper 1/2 · no calculator) (E) Solve the simultaneous equations. Show all your working.
   $x - 2y = 1$
   $xy = 3$ [5]

3. (Paper 3/4 · calculator) (E) Solve the simultaneous equations $y = x + 4$ and $x^2 + y^2 = 20$. Give your answers correct to 2 decimal places. [5]

4. (Paper 1/2 · no calculator) (E) The line $y = 6 - x$ meets the curve $xy = 8$ at the points A and B. Find the coordinates of A and B. [4]

Challenge

(E) Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 4$. Give your answers in exact (surd) form, using the quadratic formula from Ep 32, and check that each pair fits both equations.

Answers

Set A

1. $x^2 - x - 2 = 0$: (2, 4) and (−1, 1).

2. $x^2 - 3x + 2 = 0$: (1, 1) and (2, 4).

3. $x^2 - 4 = 0$: (2, 7) and (−2, −1).

4. $x(5 - x) = 6$, $x^2 - 5x + 6 = 0$: (2, 3) and (3, 2).

5. $x^2 + (x + 1)^2 = 13$, $x^2 + x - 6 = 0$: (2, 3) and (−3, −2).

6. $x^2 + (2x - 1)^2 = 2$, $5x^2 - 4x - 1 = 0$, $(5x + 1)(x - 1) = 0$: (1, 1) and (−1/5, −7/5).

7. $x^2 - 2x + 1 = 0$, $(x - 1)^2 = 0$: a repeated root, so the line is a tangent; point of contact (1, 1).

8. $x^2 + x = x - 3$ gives $x^2 + 3 = 0$; $b^2 - 4ac = 0 - 12 = -12 < 0$, so there are no real solutions and the line does not meet the curve.

Set B

1. $x^2 - 4x + 1 = 2x - 7$ M1; $x^2 - 6x + 8 = 0$, $(x - 2)(x - 4) = 0$ M1; $x = 2$, $y = -3$ A1; $x = 4$, $y = 1$ A1.

2. $x = 1 + 2y$ M1; $(1 + 2y)y = 3$, $2y^2 + y - 3 = 0$ M1; $(2y + 3)(y - 1) = 0$ M1; $y = 1$, $x = 3$ A1; $y = -3/2$, $x = -2$ A1.

3. $x^2 + (x + 4)^2 = 20$ M1; $2x^2 + 8x - 4 = 0$, $x^2 + 4x - 2 = 0$ M1; $x = -2 \pm \sqrt{6}$ M1; $x = 0.45$, $y = 4.45$ A1; $x = -4.45$, $y = -0.45$ A1.

4. $x(6 - x) = 8$ M1; $x^2 - 6x + 8 = 0$, $(x - 2)(x - 4) = 0$ M1; A(2, 4) and B(4, 2) A1 A1.

Challenge

$x^2 + (x + 1)^2 = 4$, so $2x^2 + 2x - 3 = 0$. $x = (-2 \pm \sqrt{28})/4 = (-1 \pm \sqrt{7})/2$.
$x = (-1 + \sqrt{7})/2$, $y = (1 + \sqrt{7})/2$ and $x = (-1 - \sqrt{7})/2$, $y = (1 - \sqrt{7})/2$.
Check: $x^2 + y^2 = [(8 - 2\sqrt{7}) + (8 + 2\sqrt{7})]/4 = 16/4 = 4$ for the first pair (and similarly for the second).