IM-033 · Completing the square

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 033

Tier: Extended only (E).

Set A · Fluency

1. (E) Write in the form $(x + p)^2 + q$: (a) $x^2 + 8x + 3$ (b) $x^2 - 2x + 9$

2. (E) Write $x^2 + 7x + 10$ in the form $(x + p)^2 + q$.

3. (E) Write $2x^2 + 8x - 1$ in the form $a(x + p)^2 + q$.

4. (E) Write $3x^2 - 6x + 5$ in the form $a(x + p)^2 + q$.

5. (E) Solve $x^2 + 4x - 1 = 0$ by completing the square. Give exact answers.

6. (E) Solve $x^2 - 6x - 3 = 0$ by completing the square. Give your answers in the form $p \pm q\sqrt{3}$.

7. (E) Find the coordinates of the turning point of $y = x^2 + 10x + 30$, and state whether it is a maximum or a minimum.

8. (E) Write down the turning point and the equation of the line of symmetry of $y = (x - 7)^2 - 2$.

Set B · Exam style

1. (Paper 1/2 · no calculator) (E)
   (a) Write $x^2 + 6x - 5$ in the form $(x + a)^2 + b$. [2]
   (b) Hence solve $x^2 + 6x - 5 = 0$, giving your answers in exact form. [2]

2. (Paper 1/2 · no calculator) (E) Write $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$. [3]

3. (Paper 1/2 · no calculator) (E) Show that $x^2 - 10x + 27 > 0$ for all values of x. [2]

4. (Paper 3/4 · calculator) (E)
   (a) Write $x^2 + 5x + 1$ in the form $(x + p)^2 + q$. [2]
   (b) Use your answer to part (a) to solve $x^2 + 5x + 1 = 0$. Give your answers correct to 2 decimal places. [2]

Challenge

(E) (a) Write $5 + 4x - x^2$ in the form $a - (x - b)^2$.
(b) Write down the maximum value of $5 + 4x - x^2$ and the value of x where it occurs.
(c) Use part (a) to solve $5 + 4x - x^2 = 0$, and check your answers by factorising, as in Ep 31.

Answers

Set A

1. (a) $(x + 4)^2 - 13$ (b) $(x - 1)^2 + 8$

2. $(x + 7/2)^2 - 49/4 + 40/4 = (x + 7/2)^2 - 9/4$

3. $2(x^2 + 4x) - 1 = 2[(x + 2)^2 - 4] - 1 = 2(x + 2)^2 - 9$

4. $3(x^2 - 2x) + 5 = 3[(x - 1)^2 - 1] + 5 = 3(x - 1)^2 + 2$

5. $(x + 2)^2 = 5$: $x = -2 \pm \sqrt{5}$

6. $(x - 3)^2 = 12$: $x = 3 \pm \sqrt{12} = 3 \pm 2\sqrt{3}$

7. $(x + 5)^2 + 5$: turning point $(-5, 5)$, a minimum.

8. Turning point $(7, -2)$; line of symmetry $x = 7$.

Set B

1. (a) $(x + 3)^2 - 9 - 5$ M1 $= (x + 3)^2 - 14$ A1. (b) $(x + 3)^2 = 14$ M1; $x = -3 \pm \sqrt{14}$ A1.

2. $2(x^2 - 6x) + 23$ M1; $2[(x - 3)^2 - 9] + 23$ M1; $2(x - 3)^2 + 5$ A1 ($a = 2$, $b = -3$, $c = 5$).

3. $x^2 - 10x + 27 = (x - 5)^2 + 2$ M1. $(x - 5)^2 \geq 0$, so $(x - 5)^2 + 2 \geq 2 > 0$ for all x A1.

4. (a) $(x + 5/2)^2 - 25/4 + 1$ M1 $= (x + 5/2)^2 - 21/4$ A1.
   (b) $x = -5/2 \pm \sqrt{21}/2$ M1; $x = -0.21$ or $x = -4.79$ A1.

Challenge

(a) $5 + 4x - x^2 = -(x^2 - 4x) + 5 = -[(x - 2)^2 - 4] + 5 = 9 - (x - 2)^2$.
(b) Maximum value 9, when $x = 2$.
(c) $(x - 2)^2 = 9$, $x - 2 = \pm 3$, $x = 5$ or $x = -1$. Check: $5 + 4x - x^2 = -(x^2 - 4x - 5) = -(x - 5)(x + 1)$.