IM-032 · The quadratic formula

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 032

Tier: Extended only (E). The quadratic formula is given on the Extended List of formulas.

Set A · Fluency

1. (E) Write down the values of a, b and c for each equation (rearrange first if needed).
   (a) $3x^2 - x - 7 = 0$ (b) $x^2 = 6x - 2$

2. (E) Solve $x^2 + 4x - 1 = 0$, giving your answers in surd form.

3. (E) Solve $x^2 - 6x + 4 = 0$, giving your answers in surd form.

4. (E) Solve $2x^2 + 4x - 3 = 0$, giving your answers in surd form in their simplest form.

5. (E) (calculator) Solve $x^2 + 5x + 2 = 0$, giving your answers correct to 2 decimal places.

6. (E) (calculator) Solve $3x^2 - 4x - 2 = 0$, giving your answers correct to 3 significant figures.

7. (E) Work out $b^2 - 4ac$ and state how many solutions each equation has.
   (a) $x^2 + 2x + 5 = 0$ (b) $x^2 - 10x + 25 = 0$ (c) $2x^2 + 3x - 1 = 0$

8. (E) Solve $6x^2 + x - 2 = 0$ using the formula. Then check your answers by factorising.

Set B · Exam style

1. (Paper 1/2 · no calculator) (E) Solve $x^2 - 8x + 13 = 0$. Give your answers in the form $p \pm \sqrt{q}$. [3]

2. (Paper 3/4 · calculator) (E) Solve $4x^2 + 3x - 5 = 0$. Give your answers correct to 2 decimal places. [3]

3. (Paper 3/4 · calculator) (E)
   (a) Show that the equation $(x + 1)(x - 3) = 2x$ can be written as $x^2 - 4x - 3 = 0$. [1]
   (b) Solve $x^2 - 4x - 3 = 0$. Give your answers correct to 3 significant figures. [3]

4. (Paper 1/2 · no calculator) (E) Show that the equation $3x^2 - 2x + 5 = 0$ has no real solutions. [2]

Challenge

(E) Solve $x^2 - 4x + 1 = 0$, giving your answers in surd form. Then show that one of the solutions is equal to $1/(2 + \sqrt{3})$. (Rationalise the denominator, as in Ep 21.)

Spaced review

R1. (Ep 20) Simplify $\sqrt{75} + \sqrt{12}$.

R2. (Ep 8) Work out $(3 \times 10^4) \times (5 \times 10^{-7})$. Give your answer in standard form.

Answers

Set A

1. (a) $a = 3$, $b = -1$, $c = -7$. (b) $x^2 - 6x + 2 = 0$: $a = 1$, $b = -6$, $c = 2$.

2. $b^2 - 4ac = 20$: $x = (-4 \pm 2\sqrt{5})/2 = -2 \pm \sqrt{5}$.

3. $b^2 - 4ac = 20$: $x = (6 \pm 2\sqrt{5})/2 = 3 \pm \sqrt{5}$.

4. $b^2 - 4ac = 16 + 24 = 40$, $\sqrt{40} = 2\sqrt{10}$: $x = (-4 \pm 2\sqrt{10})/4 = (-2 \pm \sqrt{10})/2$.

5. $x = (-5 \pm \sqrt{17})/2$: $x = -0.44$ or $x = -4.56$.

6. $x = (4 \pm \sqrt{40})/6$: $x = 1.72$ or $x = -0.387$.

7. (a) $4 - 20 = -16$: no real solutions. (b) $100 - 100 = 0$: one solution ($x = 5$). (c) $9 + 8 = 17$: two solutions.

8. $b^2 - 4ac = 1 + 48 = 49$: $x = (-1 \pm 7)/12$, so $x = 1/2$ or $x = -2/3$. Check: $(2x - 1)(3x + 2) = 6x^2 + x - 2$.

Set B

1. $x = (8 \pm \sqrt{(64 - 52)})/2$ M1 $= (8 \pm \sqrt{12})/2$ M1 ($\sqrt{12} = 2\sqrt{3}$) $= 4 \pm \sqrt{3}$ A1.

2. $x = (-3 \pm \sqrt{(9 + 80)})/8 = (-3 \pm \sqrt{89})/8$ M1 M1; $x = 0.80$ or $x = -1.55$ A1.

3. (a) $x^2 - 3x + x - 3 = 2x$, so $x^2 - 4x - 3 = 0$ B1.
   (b) $x = (4 \pm \sqrt{(16 + 12)})/2 = (4 \pm \sqrt{28})/2$ M1 M1; $x = 4.65$ or $x = -0.646$ A1.

4. $b^2 - 4ac = (-2)^2 - 4(3)(5) = 4 - 60$ M1 $= -56 < 0$, so there are no real solutions A1.

Challenge

$x = (4 \pm \sqrt{12})/2 = 2 \pm \sqrt{3}$. And $1/(2 + \sqrt{3}) = (2 - \sqrt{3})/((2 + \sqrt{3})(2 - \sqrt{3})) = (2 - \sqrt{3})/(4 - 3) = 2 - \sqrt{3}$, which is one of the solutions.

Spaced review

R1. $5\sqrt{3} + 2\sqrt{3} = 7\sqrt{3}$.

R2. $15 \times 10^{-3} = 1.5 \times 10^{-2}$.