IM-029 · Equations with fractions

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 029

Set A · Fluency

1. Solve $x/6 = 4$.

2. Solve $(2x − 3)/5 = 3$.

3. Solve $(x + 4)/3 = x − 2$.

4. Solve $x/2 + x/5 = 14$.

5. Solve $(x + 3)/4 − (x − 1)/6 = 1$.

6. (E) Solve $12/(x + 1) = 3$.

7. (E) Solve $x/(x − 4) = 3$.

8. (E) Solve $2/(x + 5) = 3/(2x − 1)$.

Set B · Exam style

1. (Paper 1/2 · no calculator) (C) Solve $(2x − 1)/3 + (x + 2)/4 = 2$. [3]

2. (Paper 3/4 · calculator) (C) Solve $(x + 2.6)/4.5 = (x − 1.2)/2.8$. Give your answer correct to 3 significant figures. [3]

3. (Paper 1/2 · no calculator) (E) Solve $4/(x − 3) = 7/(2x + 1)$. [3]

4. (Paper 1/2 · no calculator) (E)
   (a) Show that the equation $6/(x + 4) + 1/(2x + 1) = 1$ can be written as $x^2 − 2x − 3 = 0$. [3]
   (b) Solve the equation $6/(x + 4) + 1/(2x + 1) = 1$. [2]

Challenge

(E · links to Ep 26, the difference of two squares, and Ep 20, surds) Solve $x/(x − 1) − 1/(x + 1) = 4/(x^2 − 1)$. Give your answers in surd form. (Hint: $x^2 − 1 = (x + 1)(x − 1)$.)

Answers

Set A

1. $x = 24$

2. $2x - 3 = 15$, $x = 9$

3. $x + 4 = 3x - 6$, $x = 5$

4. $5x + 2x = 140$, $x = 20$

5. $3(x + 3) - 2(x - 1) = 12$, $x + 11 = 12$, $x = 1$

6. $12 = 3(x + 1)$, $x = 3$

7. $x = 3(x - 4)$, $x = 6$

8. $2(2x - 1) = 3(x + 5)$, $4x - 2 = 3x + 15$, $x = 17$

Set B

1. × 12: $4(2x - 1) + 3(x + 2) = 24$ M1; $8x - 4 + 3x + 6 = 24$, $11x + 2 = 24$ M1; $x = 2$ A1.

2. $2.8(x + 2.6) = 4.5(x - 1.2)$ M1; $2.8x + 7.28 = 4.5x - 5.4$, so $12.68 = 1.7x$ M1; $x = 7.458\ldots = 7.46$ A1.

3. $4(2x + 1) = 7(x - 3)$ M1; $8x + 4 = 7x - 21$ M1; $x = -25$ A1.

4. (a) × $(x + 4)(2x + 1)$: $6(2x + 1) + (x + 4) = (x + 4)(2x + 1)$ M1; $13x + 10 = 2x^2 + 9x + 4$ A1; $2x^2 - 4x - 6 = 0$, so $x^2 - 2x - 3 = 0$ A1.
   (b) $(x - 3)(x + 1) = 0$ M1; $x = 3$ or $x = -1$ A1 (neither makes a denominator zero: those values are -4 and $-1/2$).

Challenge

Multiply every term by $(x + 1)(x - 1)$: $x(x + 1) - (x - 1) = 4$, so $x^2 + x - x + 1 = 4$ and $x^2 = 3$.
$x = \sqrt{3}$ or $x = -\sqrt{3}$. Neither is 1 or −1, so both are allowed.