IM-023 · Sets II: Venn diagrams with two and three sets

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 023

(For a two-set Venn diagram, call the regions: A only, both, B only, neither.)

Set A · Fluency

1. For two sets A and B, which regions make up (a) $A \cap B'$ (b) $(A \cup B)'$ (c) $A' \cup B$? Shade each on a Venn diagram.

2. Write in set notation: (a) the region in both A and B (b) the region "B only" (c) the regions in exactly one of the sets.

3. 35 people were asked about drinks. 21 like tea, 17 like coffee and 9 like both. How many like neither?

4. 28 students: 16 play chess, 13 play draughts and 4 play neither. How many play both?

5. n(A) = 18, n(B) = 11 and n(A ∪ B) = 24. Find n(A ∩ B).

6. 60 people were asked about cities they have visited. 35 have been to Paris, 22 have been to Rome, 12 have been to Rome but not Paris. Find (a) the number who have been to both (b) the number who have been to neither.

7. (E) $\mathcal{E}$ = {1, 2, …, 12}, A = {1, 2, 3, 4, 5, 6}, B = {even numbers}, C = {multiples of 3}. Draw a three-set Venn diagram showing every element of $\mathcal{E}$.

8. (E) Using question 7, list $A \cap (B \cup C)'$ and find $n((A \cup B \cup C)')$.

Set B · Exam style

1. (Paper 1/2 · no calculator) (C) In a class of 45, 26 students have a laptop (L), 30 have a tablet (T) and 6 have neither.
   (a) Find $n(L \cap T)$. [2]
   (b) Draw a Venn diagram showing the number of students in each region. [1]
   (c) How many students have exactly one of the two? [1]

2. (Paper 1/2 · no calculator) (C) Use set notation to describe each region.
   (a) The region inside circle B but outside circle A, together with the region outside both circles. [1]
   (b) The overlap of A and B, together with the region outside both circles. [1]

3. (Paper 2 · no calculator) (E) $n(\mathcal{E}) = 50$, $n(A) = 2x + 3$, $n(B) = x + 8$, $n(A \cap B) = x$ and $n((A \cup B)') = 9$. Find the value of x. [3]

4. (Paper 2 · no calculator) (E) 70 people were asked whether they enjoy reading (R), music (M) and sport (S).
   $n(R) = 30$, $n(M) = 33$, $n(S) = 28$, $n(R \cap M) = 10$, $n(M \cap S) = 9$, $n(R \cap S) = 7$, $n(R \cap M \cap S) = 4$.
   (a) Draw a Venn diagram showing the number in each region. [3]
   (b) How many enjoy none of the three? [1]

Challenge

(E) In a school, 60% of the students study Physics, 45% study Chemistry and 20% study neither. (a) What percentage study both? (b) 150 students study both. How many students are in the school? (Links to Ep 16 and Ep 17, percentages.)

Answers

Set A

1. (a) A only (b) neither (c) both, B only and neither (everything except A only).

2. (a) $A \cap B$ (b) $A' \cap B$ (c) $(A \cap B') \cup (A' \cap B)$, or $(A \cup B) \cap (A \cap B)'$.

3. $n(T \cup C) = 21 + 17 - 9 = 29$, so neither $= 35 - 29 = 6$.

4. $n(C \cup D) = 28 - 4 = 24$; both $= 16 + 13 - 24 = 5$.

5. $18 + 11 - 24 = 5$.

6. (a) $22 - 12 = 10$ (b) Paris only 25; $60 - (25 + 10 + 12) = 13$.

7. $A \cap B \cap C$: 6; $A \cap B$ only: 2, 4; $A \cap C$ only: 3; $B \cap C$ only: 12; A only: 1, 5; B only: 8, 10; C only: 9; none: 7, 11.

8. $A \cap (B \cup C)' = \{1, 5\}$; $n((A \cup B \cup C)') = 2$.

Set B

1. (a) $n(L \cup T) = 45 - 6 = 39$ M1; $26 + 30 - 39 = 17$ A1. (b) L only 9, both 17, T only 13, neither 6 B1. (c) $9 + 13 = 22$ B1.

2. (a) A' B1 (b) $(A \cap B) \cup (A \cup B)'$ (or equivalent, e.g. $(A \cap B')' \cap (A' \cap B)'$) B1.

3. A only $x + 3$, B only 8 M1; $(x + 3) + x + 8 + 9 = 50$ M1; $2x + 20 = 50$, $x = 15$ A1.

4. (a) Centre 4; $R \cap M$ only 6, $M \cap S$ only 5, $R \cap S$ only 3 B1; R only 17, M only 18, S only 16 B1 B1 (B1 for two of three). (b) $70 - 69 = 1$ B1.

Challenge

(a) Physics or Chemistry: $100\% - 20\% = 80\%$; both $= 60\% + 45\% - 80\% = 25\%$.
(b) 25% is 150, so 100% is $150 \times 4 = 600$ students.