IM-001 · Types of number

IGCSE Mathematics (0580) · HomiLabs: Maths · Ep 001

All questions are Core (C) unless tagged (E).

Set A · Fluency

1. From the list $-6,\ 0,\ 4,\ 3.5,\ 11,\ -\frac{1}{2}$, write down (a) the natural numbers (b) the integers.

2. Is 79 prime? Show how you decide.

3. Write down all the prime numbers between 30 and 50.

4. From the list $4,\ 8,\ 10,\ 25,\ 27,\ 49,\ 100,\ 125$, write down (a) the square numbers (b) the cube numbers.

5. Write down the reciprocal of (a) 9 (b) $\frac{5}{6}$ (c) -0.5.

6. Rational or irrational? Give a reason for each. (a) $\sqrt{49}$ (b) $\sqrt{7}$ (c) 0.35 (d) 0.272727… (the digits 27 repeat forever)

7. Write in figures: (a) five million, thirty thousand and eight (b) two billion.

8. Write in words: (a) 70 015 (b) 3 200 400.

Set B · Exam style

1. (Paper 1/2 · no calculator) (C) Here is a list of numbers: $2,\ 9,\ 21,\ 31,\ 51,\ 64,\ \sqrt{16},\ \sqrt{11}$.
   From the list, write down
   (a) the two prime numbers [1]
   (b) the three square numbers [1]
   (c) the irrational number. [1]

2. (Paper 1/2 · no calculator) (C) (a) Explain why 1 is not a prime number. [1] (b) Explain why 2 is a prime number. [1]

3. (Paper 1/2 · no calculator) (C) (a) Find the reciprocal of $1\frac{1}{4}$. [2]
   (b) Show that 0.8 multiplied by its reciprocal is 1. [1]

4. (Paper 3/4 · calculator) (C) (a) Use your calculator to work out $\sqrt{2} \times \sqrt{8}$. [1]
   (b) Is your answer to part (a) rational or irrational? Give a reason. [1]

Challenge

(E) The number 1 is both a square number and a cube number, and so is 64. Find the next whole number after 64 that is both a square number and a cube number. Explain how you know there is none in between.

Answers

Set A

1. (a) 4, 11 (b) −6, 0, 4, 11

2. 79 is odd (not 2); 7 + 9 = 16 (not 3); it does not end in 0 or 5 (not 5); 79 = 7 × 11 + 2 (not 7). So 79 is prime.

3. 31, 37, 41, 43, 47

4. (a) 4, 25, 49, 100 (b) 8, 27, 125

5. (a) $\frac{1}{9}$ (b) $\frac{6}{5}$ (c) $-0.5 = -\frac{1}{2}$, so the reciprocal is −2

6. (a) $\sqrt{49} = 7 = \frac{7}{1}$: rational (b) irrational (7 is not a square number) (c) $0.35 = \frac{35}{100} = \frac{7}{20}$: rational (d) rational: the decimal repeats (it equals $\frac{3}{11}$)

7. (a) 5 030 008 (b) 2 000 000 000

8. (a) seventy thousand and fifteen (b) three million, two hundred thousand, four hundred

Set B

1. (a) 2, 31 B1 (21 = 3 × 7 and 51 = 3 × 17 are not prime) (b) 9, 64, $\sqrt{16}$ (= 4) B1 (c) $\sqrt{11}$ B1

2. (a) 1 has only one factor; a prime has exactly two factors B1 (b) 2 has exactly two factors, 1 and 2 B1

3. (a) $1\frac{1}{4} = \frac{5}{4}$ M1, reciprocal $\frac{4}{5}$ (or 0.8) A1 (b) $0.8 = \frac{4}{5}$, reciprocal $\frac{5}{4}$, and $\frac{4}{5} \times \frac{5}{4} = \frac{20}{20} = 1$ B1 (or $0.8 \times 1.25 = 1$)

4. (a) 4 B1 (b) Rational, because $4 = \frac{4}{1}$ can be written as a fraction B1

Challenge

A number that is both a square and a cube is a whole number to the power 6: $1^6 = 1$, $2^6 = 64$, $3^6 = 729$. So the next one is 729 = 27² = 9³. Check between: the cubes after 64 are 125, 216, 343, 512, 729, and of these only 729 is a square (125, 216, 343 and 512 lie strictly between consecutive squares 11² = 121 and 12² = 144, 14² = 196 and 15² = 225, 18² = 324 and 19² = 361, 22² = 484 and 23² = 529).